

# Quantum-accelerated scientific computing: *concepts, programming tools and applications*

Matthias Möller

Delft University of Technology  
Delft Institute of Applied Mathematics



Joint work with Boyang Chen, Marc Gerritsma, Zaid Al-Ars, Jens de Zoete, Giorgio Balducci, Enrico Cappanera, Smaran Adarsh, Merel Schalkers, Koen Mesman, Tim Driebergen, ...

# Quantum computing in the news

## Article

### Quantum supremacy using a programmable superconducting processor

<https://doi.org/10.1038/s41586-019-1666-5>

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Frank Arute<sup>1</sup>, Kunal Arya<sup>1</sup>, Ryan Babbush<sup>1</sup>, Dave Bacon<sup>1</sup>, Joseph C. Bardin<sup>1,2</sup>, Rami Barends<sup>1</sup>, Rupak Biswas<sup>3</sup>, Sergio Boixo<sup>1</sup>, Fernando G. S. L. Brandao<sup>1,4</sup>, David A. Buell<sup>1</sup>, Brian Burkett<sup>1</sup>, Yu Chen<sup>1</sup>, Zijun Chen<sup>1</sup>, Ben Chiaro<sup>5</sup>, Roberto Collins<sup>1</sup>, William Courtney<sup>1</sup>, Andrew Dunsworth<sup>1</sup>, Edward Farhi<sup>1</sup>, Brooks Foxen<sup>1,5</sup>, Austin Fowler<sup>1</sup>, Craig Gidney<sup>1</sup>, Marissa Giustina<sup>1</sup>, Rob Graff<sup>1</sup>, Keith Guerin<sup>1</sup>, Steve Habegger<sup>1</sup>, Matthew P. Harrigan<sup>1</sup>, Michael J. Hartmann<sup>1,6</sup>, Alan Ho<sup>1</sup>, Markus Hoffmann<sup>1</sup>, Trent Huang<sup>1</sup>, Travis S. Humble<sup>7</sup>, Sergei V. Isakov<sup>1</sup>, Evan Jeffrey<sup>1</sup>,



October 21, 2019 | Written by: Edwin Pednault, John Gunnels & Dmitri Maslov, and Jay Gambetta

Recent advances in quantum computing have resulted in two 53-qubit processors: one from our group in IBM and a device described by Google in a paper published in the journal *Nature*. In the paper, it is argued that their device reached “quantum supremacy” and that “a state-of-the-art supercomputer **would require approximately 10,000 years** to perform the equivalent task.” ***We argue that an ideal simulation of the same task can be performed on a classical system in 2.5 days and with far greater fidelity.*** This is in fact a conservative, worst-case estimate, and we expect that with additional refinements the classical cost of the simulation can be further reduced.

A screenshot of a TechWire Asia article. The title is "The quantum computing race is heating up as the Chinese surpass Google". The author is Dashveer Kaur, dated 20 July, 2021. There are social media icons for Facebook, Twitter, and LinkedIn. A list of bullet points follows: "China is unveiling a super-advanced 66-qubit quantum supercomputer called 'Zuchongzhi'" and "The Chinese team claims that it has solved a problem in just over an hour that would otherwise take the world's most powerful classical supercomputer eight years to crack." There is a "56 SOCIAL BUZZ" badge and a small image of a person working with quantum hardware.

TECHWIRE ASIA Insights Latest Popular

### The quantum computing race is heating up as the Chinese surpass Google

56 SOCIAL BUZZ

Chinese quantum computer surpasses Google's 'quantum supremacy'.

By Dashveer Kaur | 20 July, 2021

- China is unveiling a super-advanced 66-qubit quantum supercomputer called “Zuchongzhi”
- The Chinese team claims that it has solved a problem in just over an hour that would otherwise take the world’s most powerful classical supercomputer eight years to crack.

# Quantum computing in Europe



Computertechnologie

## Ein Quantensprung für Deutschland?

Stand: 15.06.2021 18:25 Uhr

In Ehningen bei Stuttgart wurde Europas erster Quantencomputer eingeweiht. Der ultraschnelle Rechner der Firma IBM soll der Wirtschaft helfen, in Wettstreit mit China und den USA zu bestehen.

TECHNOLOGY NEWS MAY 11, 2021 / 10:52 AM / UPDATED 5 MONTHS AGO

## Germany to support quantum computing with 2 billion euros

By Reuters Staff

2 MIN READ



## THE FIRST EUROPEAN ONLINE QUANTUM COMPUTER PLATFORM

4 May 2020 • 3 min reading time

Leading universities and quantum hubs from China to America and the Netherlands are working on the development of a usable quantum computer. Within QuTech, TNO is working on innovative quantum technology in collaboration with Delft University of Technology and with some success, because a new version of the 'Quantum Inspire' quantum computing platform was launched on 20 April 2020. It is, in fact, the first European quantum computer platform that is generally accessible online.

09.04.2021. Awards

Quantum Delta NL awarded 615 million euro from Netherlands' National Growth Fund to accelerate quantum technology



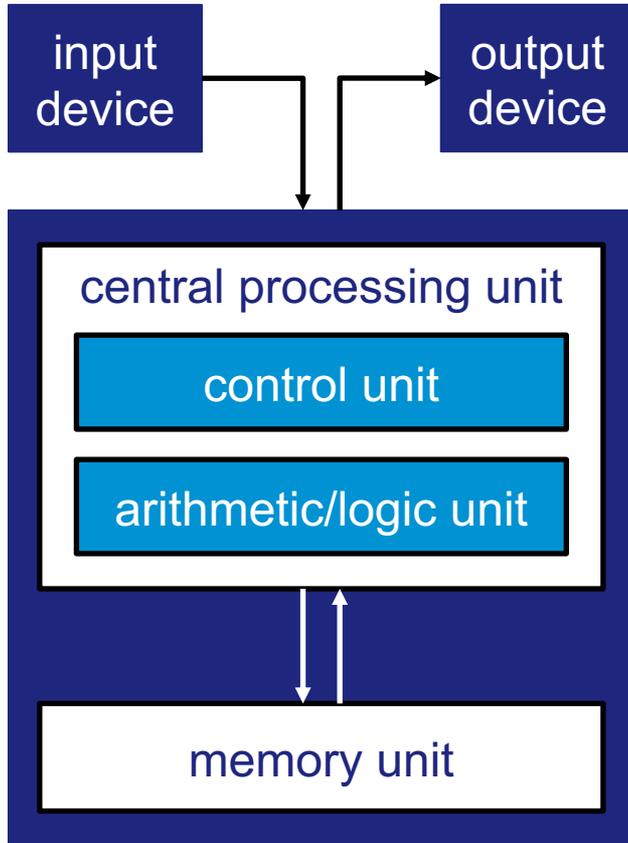
# Quantum-accelerated scientific computing

- **concepts**
  - *qubits, gates, and simple algorithms*
- **programming tools**
  - *LibKet and generation of resource-optimal quantum circuits*
- **applications**
  - *quantum linear solvers and optimization algorithms*
- Conclusion

# concepts

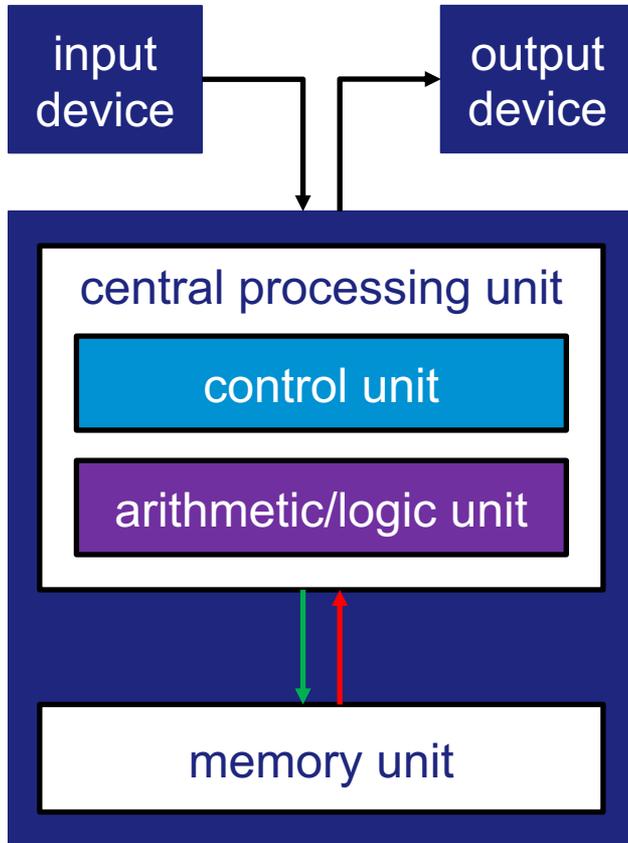
*qubits, gates, and simple algorithms*

# von Neumann model



```
int a = 1;  
int b = 2;  
int c = a+b;
```

# von Neumann model

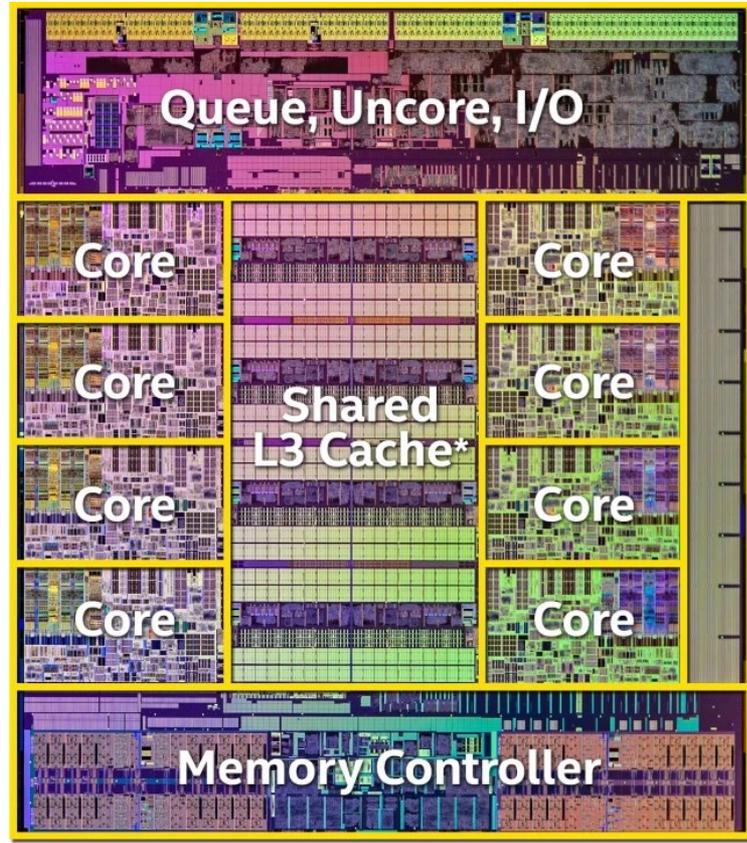
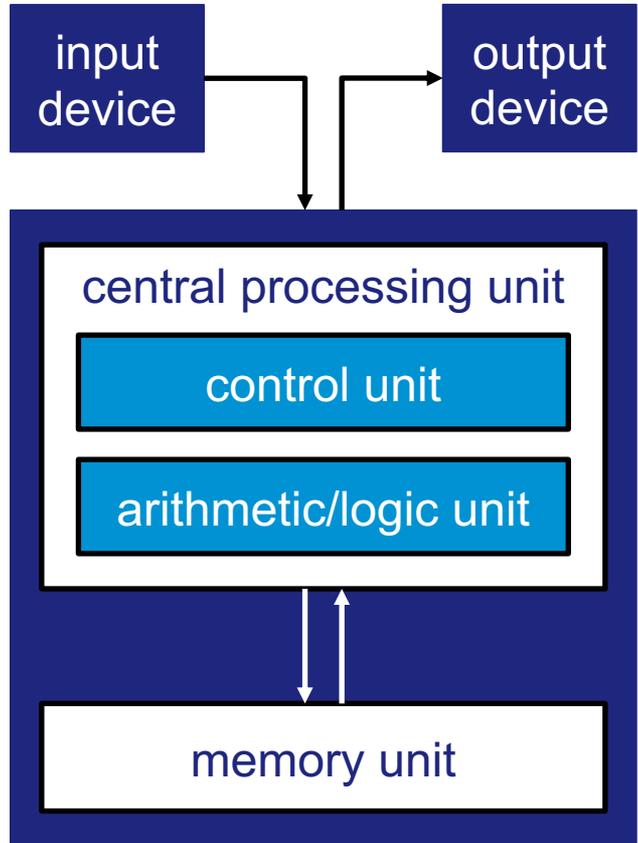


```
int a = 1;
int b = 2;
int c = a+b;
```

```
ld r0 mem(a)
ld r1 mem(b)
add r0 r1 r2
sd r2 mem(c)
```

```
10001100000010100000110000100000
10001100010010110000001001100010
101011011000101000000010110100110
100001001000101000000010000010011
```

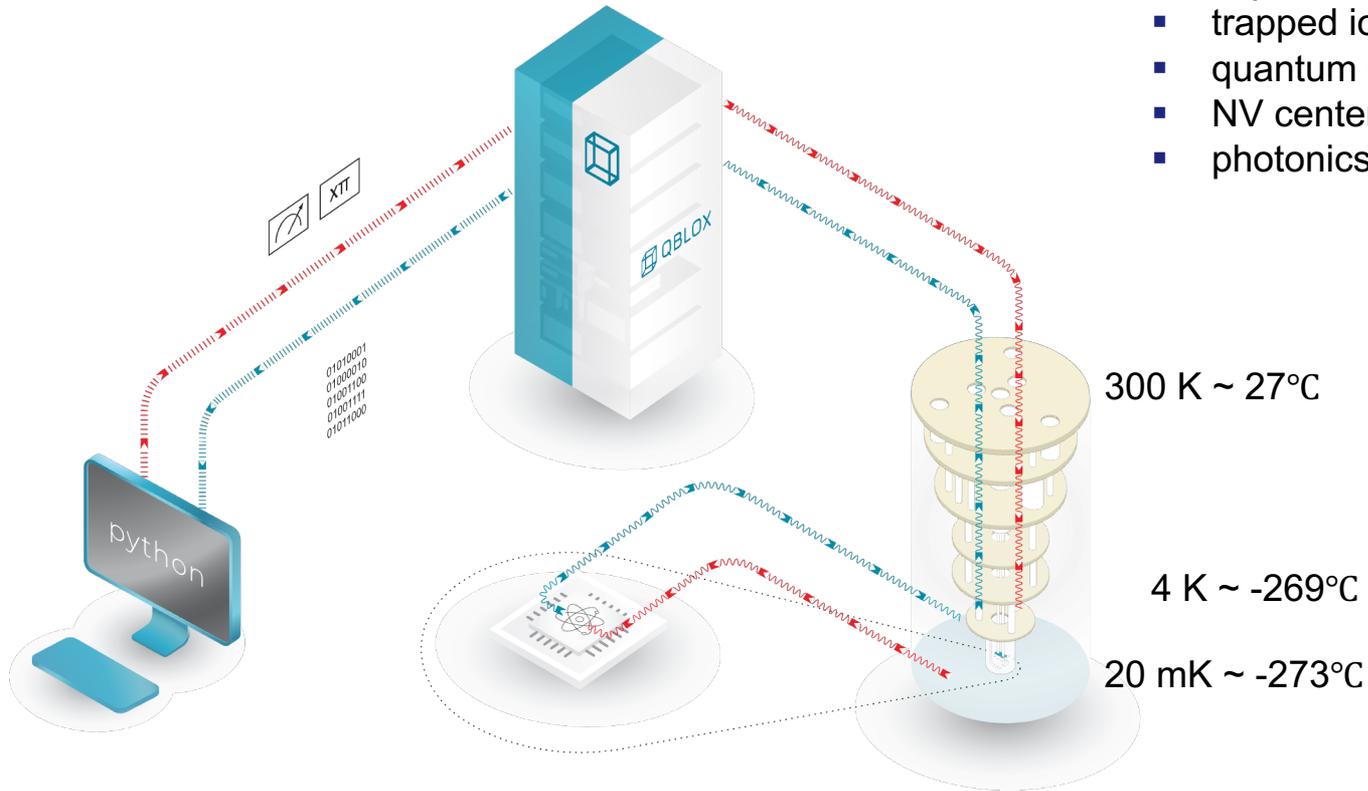
# von Neumann model



Intel® Core™ i7-3960X Processor

# a quantum computer model

- superconducting
- trapped ion
- quantum dots
- NV centers
- photonics (room temperature)

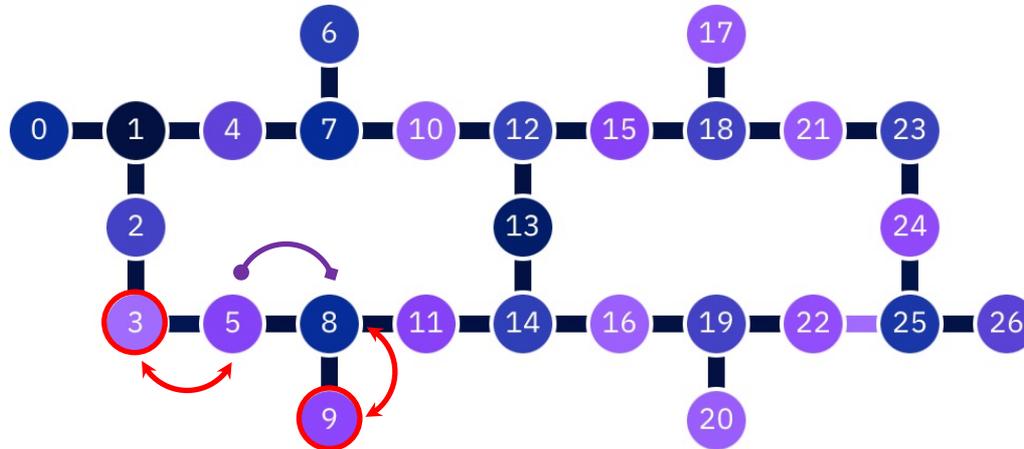


# IBM's 27-qubit processor

data is 'stored' in qubits  
and can be manipulated  
by 1- and 2-qubit 'gates'

controlled-NOT gate  
between q3 and q9

swap q3 q5  
swap q9 q8  
cnot q5 q8



2-qubit gates between nonadjacent  
qubits require additional 'swap' ops

# quantum bits

- **qubit**: quantum version of a bit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad \alpha, \beta \in \mathbb{C}, \quad |\alpha|^2 + |\beta|^2 = 1$$

- computational **basis**

$$\mathcal{E} = (|0\rangle, |1\rangle) = \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

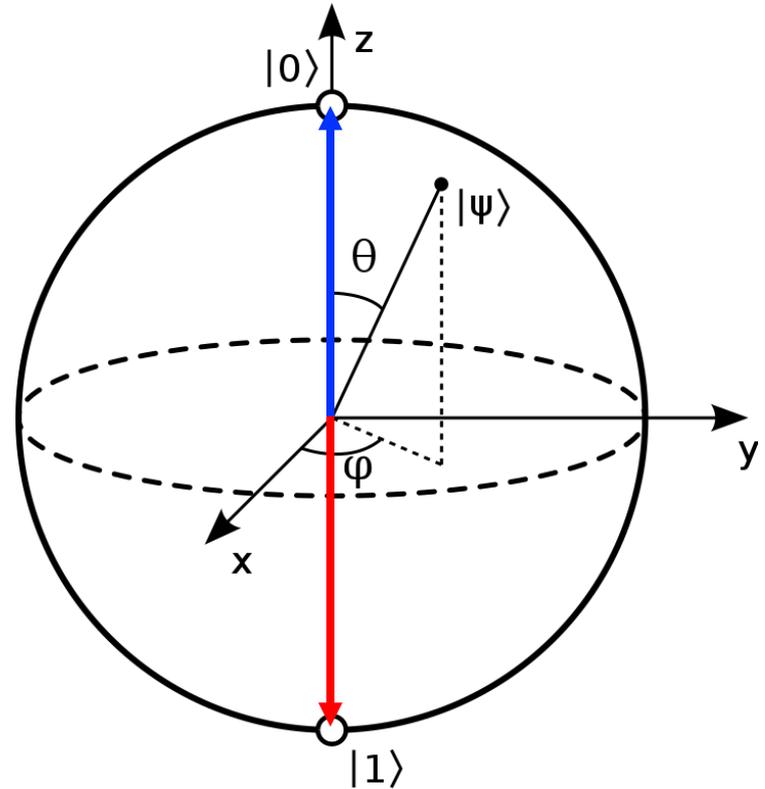
- coefficients  $\alpha, \beta$  are the **probability amplitudes** and  $|\alpha|^2$  and  $|\beta|^2$  are the **probabilities** of measuring the basis states  $|0\rangle$  and  $|1\rangle$ , respectively

# single-qubit states

- **Bloch sphere**

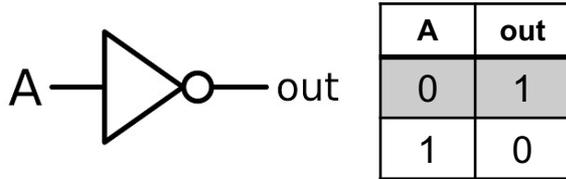
$$|\psi\rangle = \cancel{e^{i\delta}} \left( \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle \right)$$

- polar angle  $\theta \in [0, \pi]$
- azimuthal angle  $\varphi \in [0, 2\pi)$
- ~~global phase  $\delta$~~

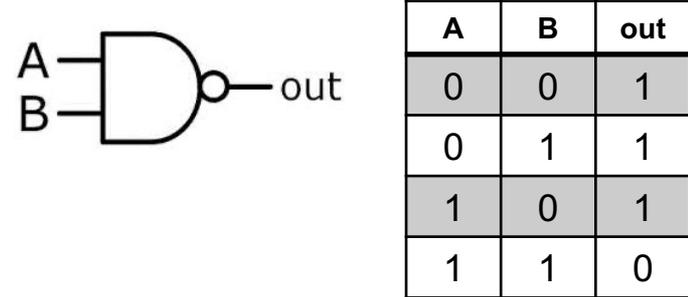


# classical gates

- **NOT**



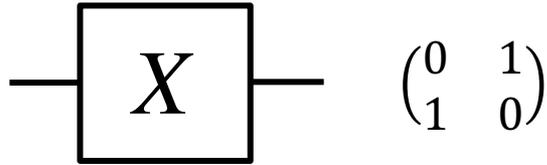
- **NAND**



- logical operations based on truth tables
- most classical gates are not reversible

# quantum gates

- **Pauli X**

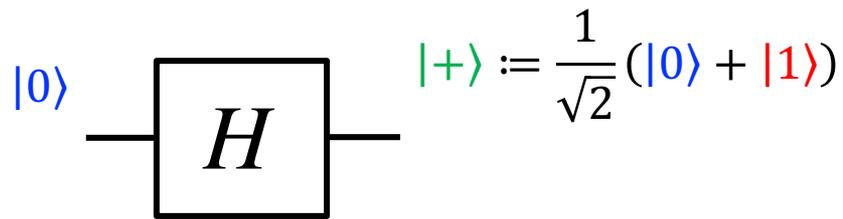
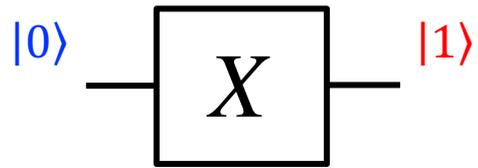


- **Hadamard**

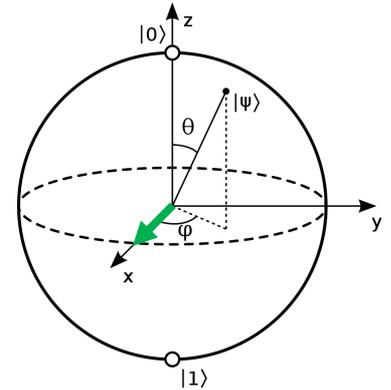
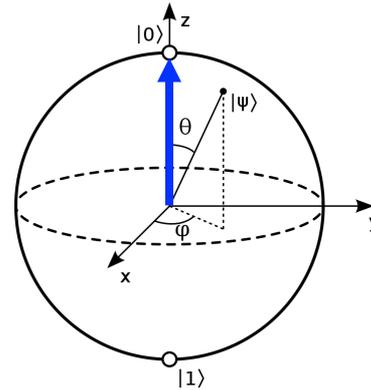
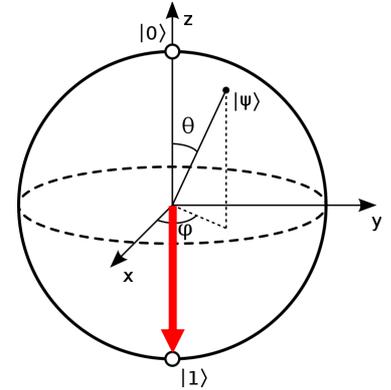
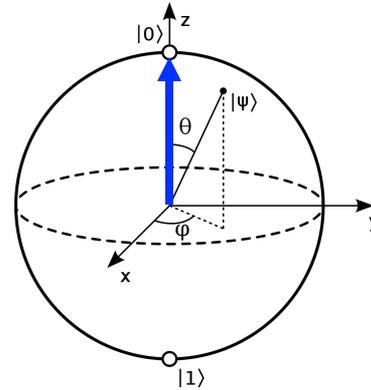


- unitary operations represented by unitary matrices
- all quantum gates are reversible, e.g.  $HH^\dagger = I$

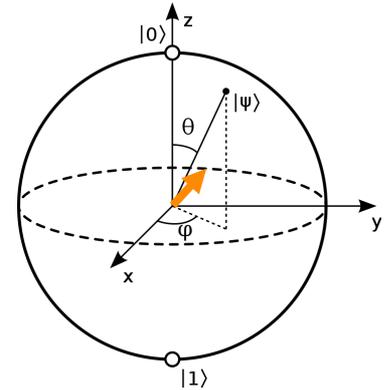
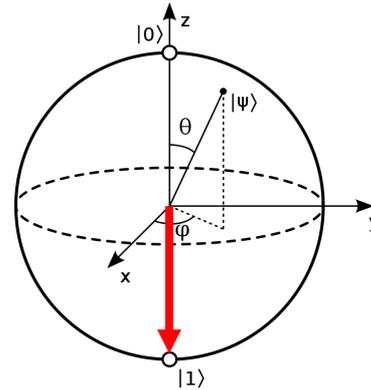
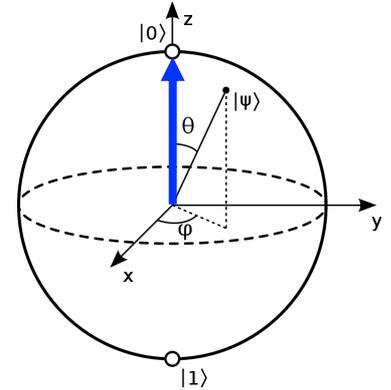
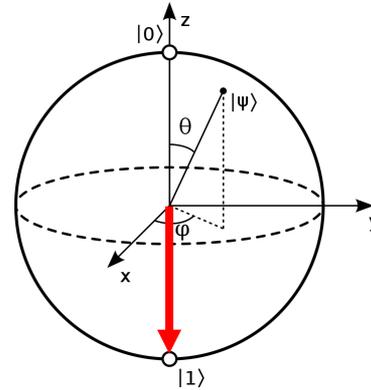
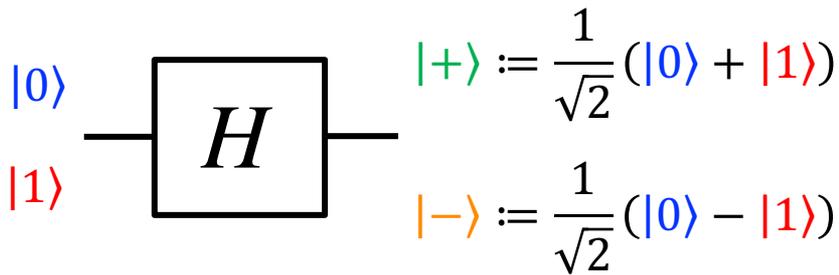
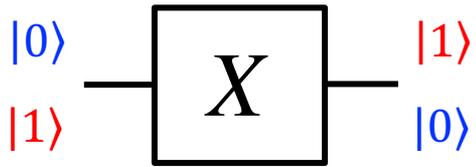
# single-qubit gates



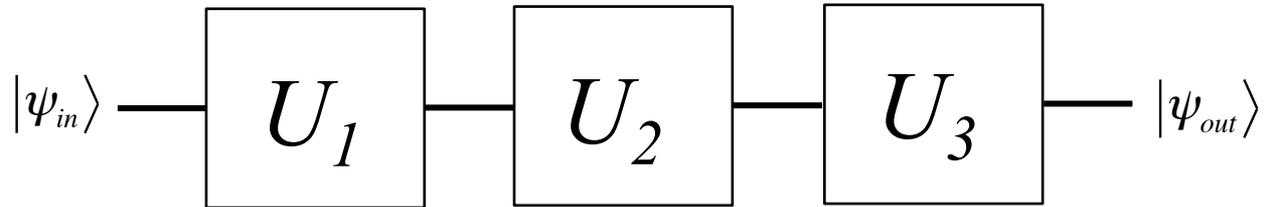
$$|+\rangle := \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$



# single-qubit gates



## single-qubit circuits



- single-qubit gates  $U_k$  are **unitary matrices**, i.e.

$$U_k U_k^\dagger = U_k^\dagger U_k = I$$

- quantum circuits are sequences of matrix-vector multiplications

$$|\psi_{out}\rangle = U_3 U_2 U_1 |\psi_{in}\rangle$$

## multi-qubit states

- $|\psi_0\rangle = \alpha_0|0\rangle + \beta_0|1\rangle = \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta_0 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

tensor product

- $|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle = \alpha_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix}$$

- **tensor product** of two single-qubit states

$$|\psi_0\rangle \otimes |\psi_1\rangle = \alpha_0\alpha_1|00\rangle + \alpha_0\beta_1|01\rangle + \beta_0\alpha_1|10\rangle + \beta_0\beta_1|11\rangle =: |\psi_0\psi_1\rangle$$

with

$$|\alpha_0\alpha_1|^2 + |\alpha_0\beta_1|^2 + |\beta_0\alpha_1|^2 + |\beta_0\beta_1|^2 = 1$$

# multi-qubit states

- **tensor product** of  $n$  single-qubit states

$$|\psi_0 \dots \psi_n\rangle = \gamma_{0\dots 00} |0 \dots 00\rangle + \gamma_{0\dots 01} |0 \dots 01\rangle + \dots + \gamma_{1\dots 11} |1 \dots 11\rangle$$

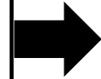
- an  $n$ -qubit register can hold the  $2^n$  inputs ‘simultaneously’ in **superposition**
- **a few words of caution**
  - it is impossible to obtain the  $\gamma$ 's; one obtains a single binary answer, say,  $|001101\rangle$  with probability  $|\gamma_{001101}|^2$  upon **measurement**
  - a single run of a quantum circuit is not very useful; many runs are required to measure the correct answer with sufficient certainty

# example: 3-bit password

**classical:**

000  
001  
010  
011  
100  
101  
110  
111

password:  
\_\_\_\_\_

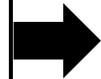


~~010~~ ✓

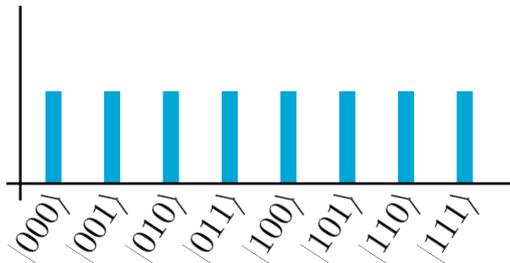
**quantum:**

$|000\rangle$   
 $|001\rangle$   
 $|010\rangle$   
 $|011\rangle$   
 $|100\rangle$   
 $|101\rangle$   
 $|110\rangle$   
 $|111\rangle$

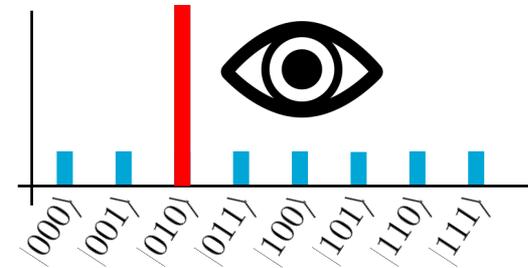
password:  
\_\_\_\_\_



$|010\rangle$  ✓

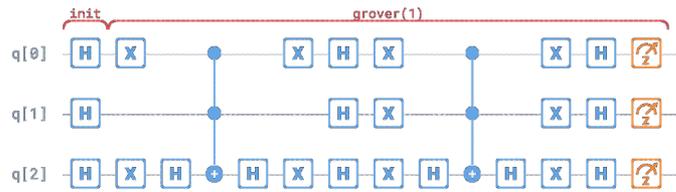


Grover's  
algorithm

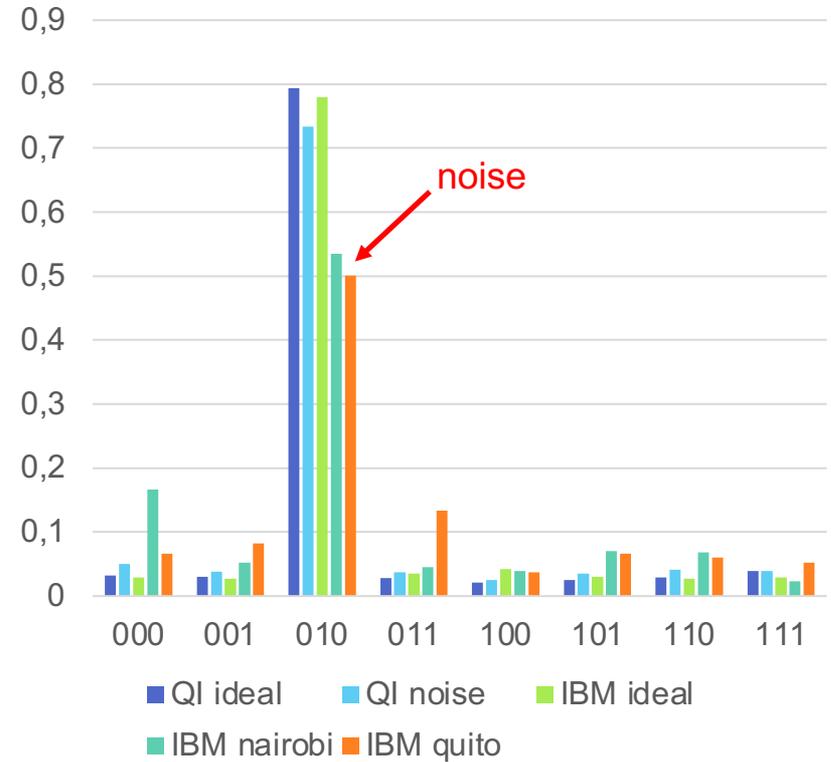
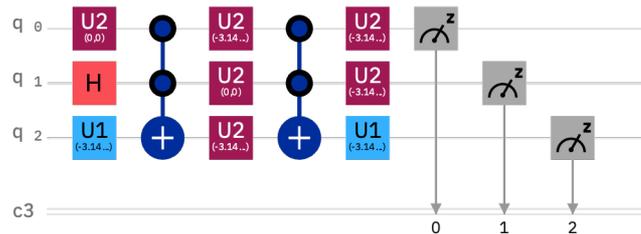


# Grover's algorithm

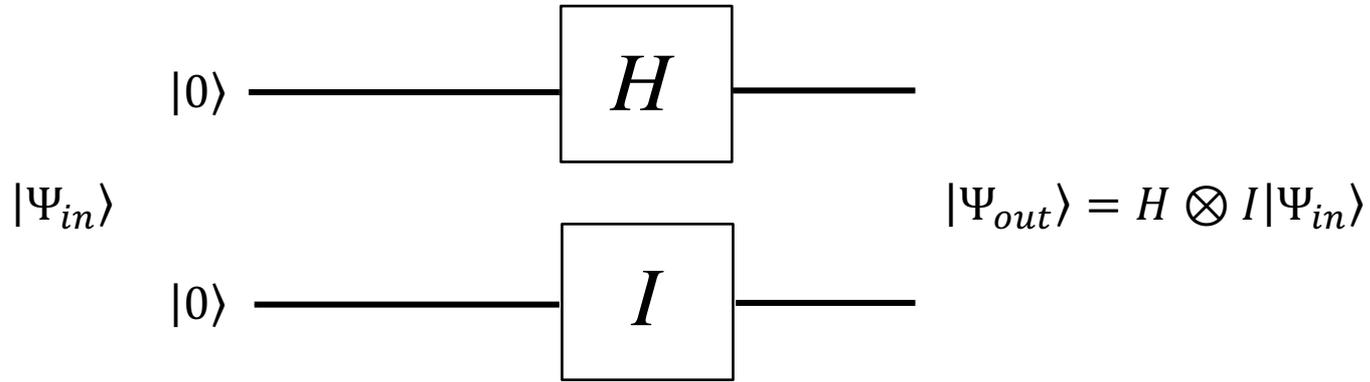
- quantum circuit on QI



- quantum circuit on IBM



## multi-qubit gates

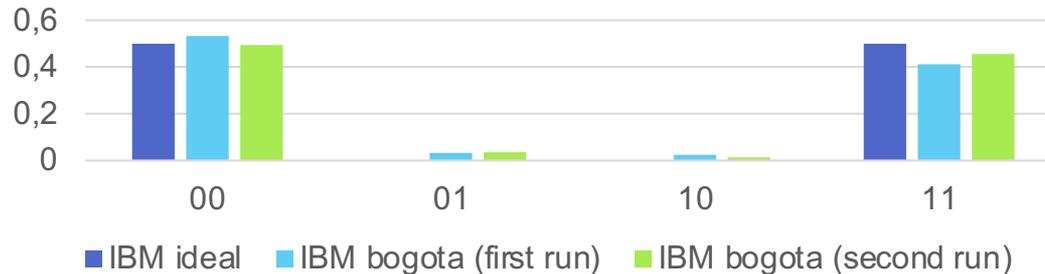


$$H \otimes I |00\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{|00\rangle + |10\rangle}{\sqrt{2}} = \frac{(|0\rangle + |1\rangle) \otimes |0\rangle}{\sqrt{2}}$$

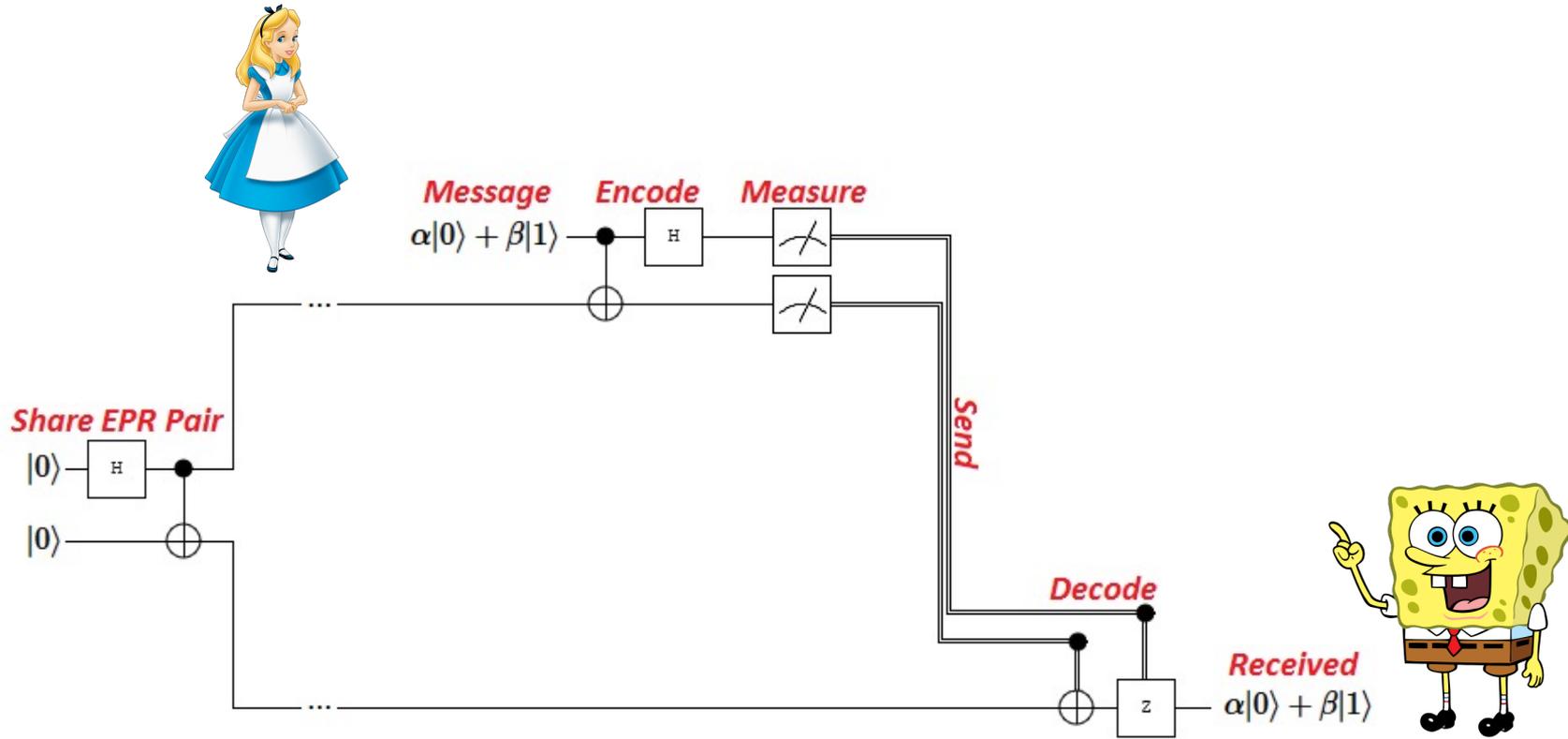
# entanglement

$$CNOT(H \otimes I)|00\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

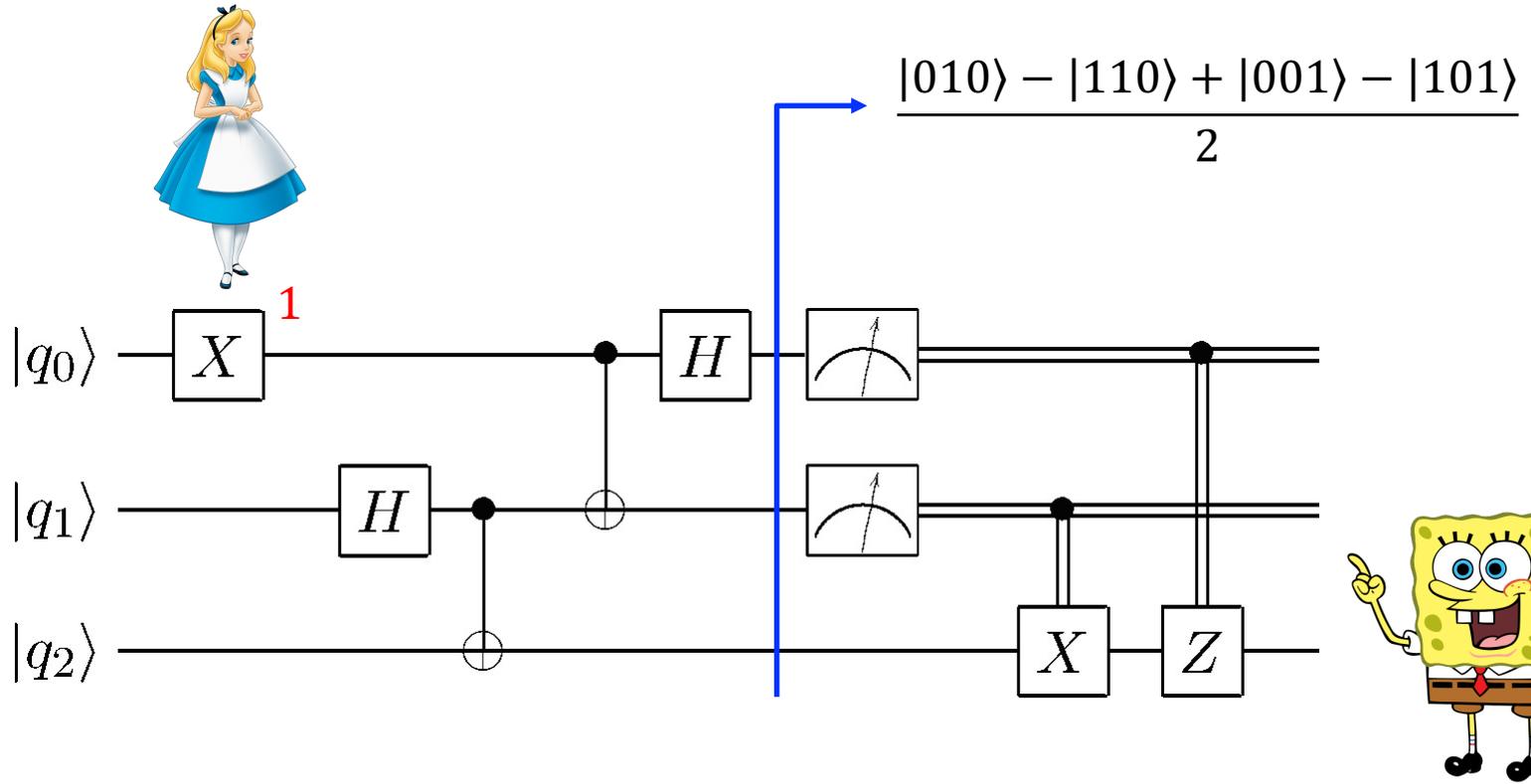
- Bell state is maximally entangled. By measuring one of the two qubits one knows the value of the other qubit without a further measurement



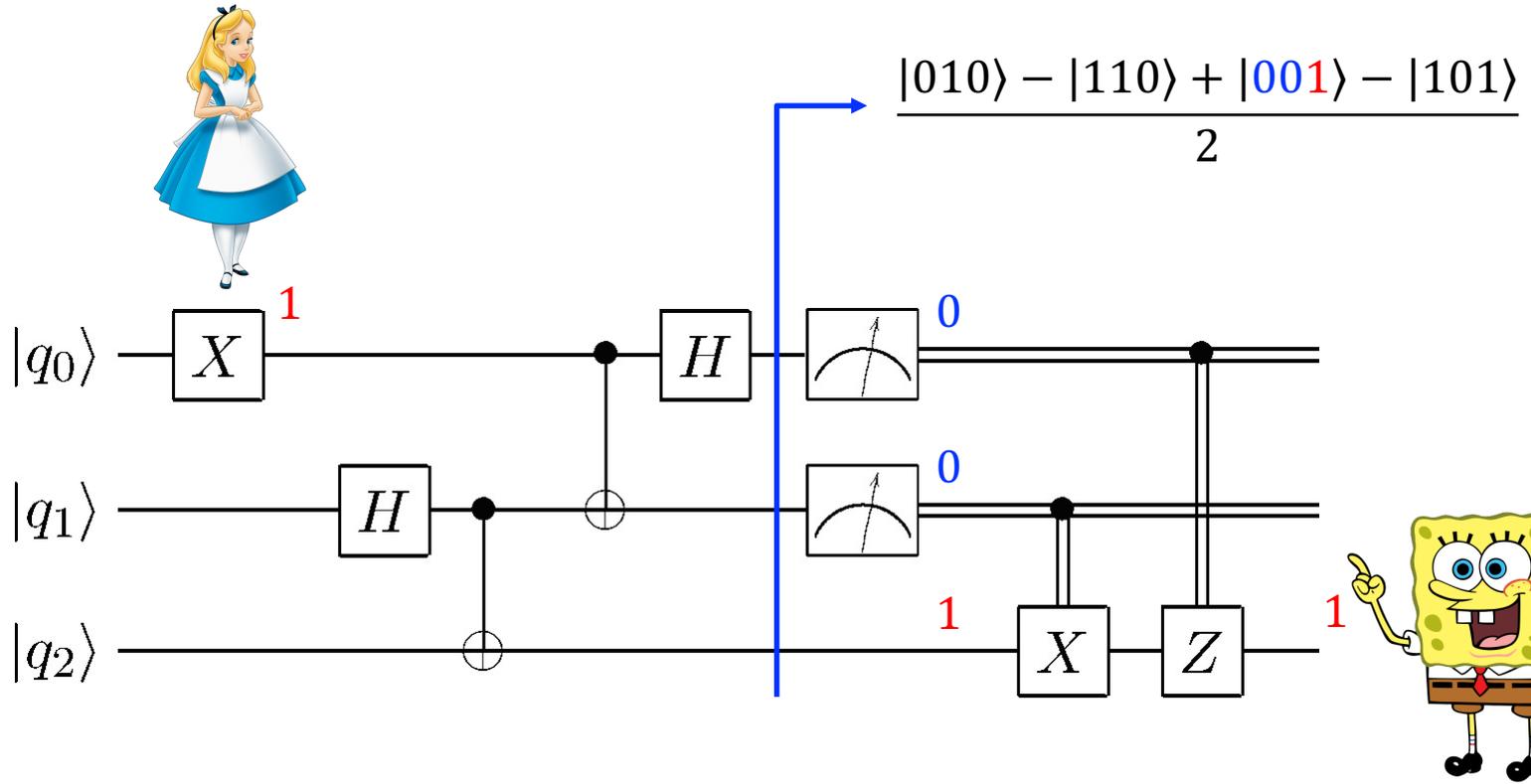
# quantum teleportation



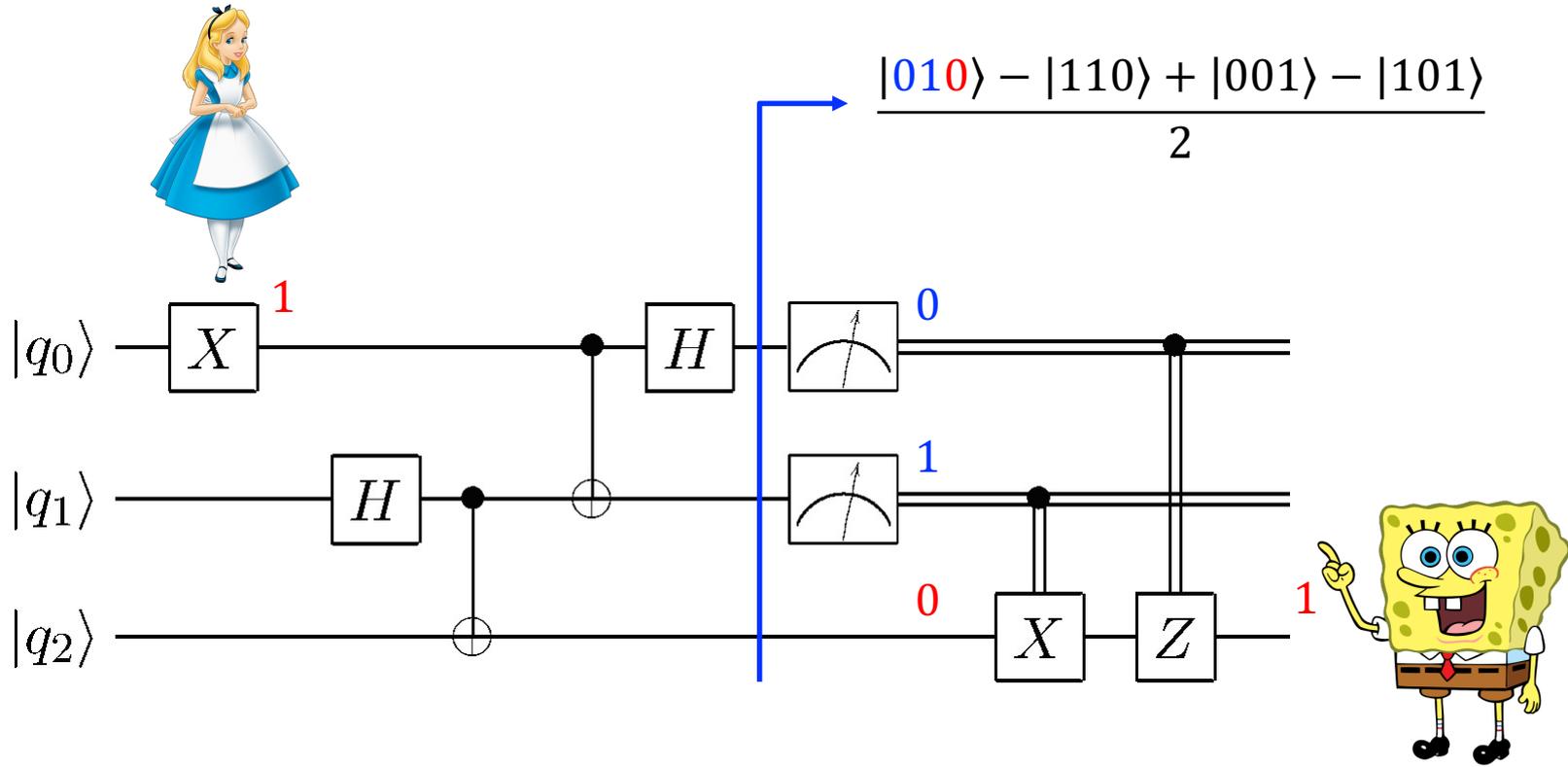
# quantum teleportation



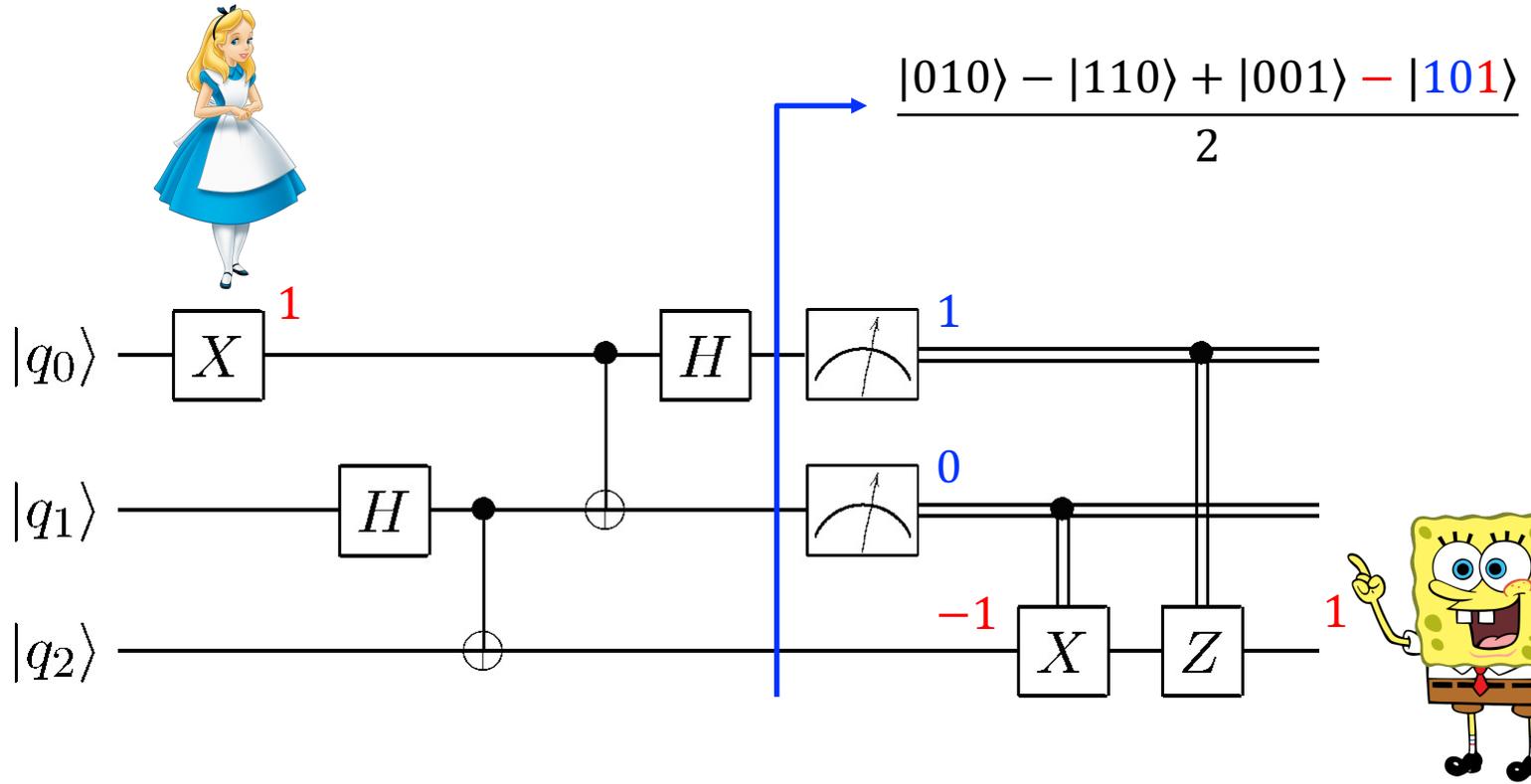
# quantum teleportation



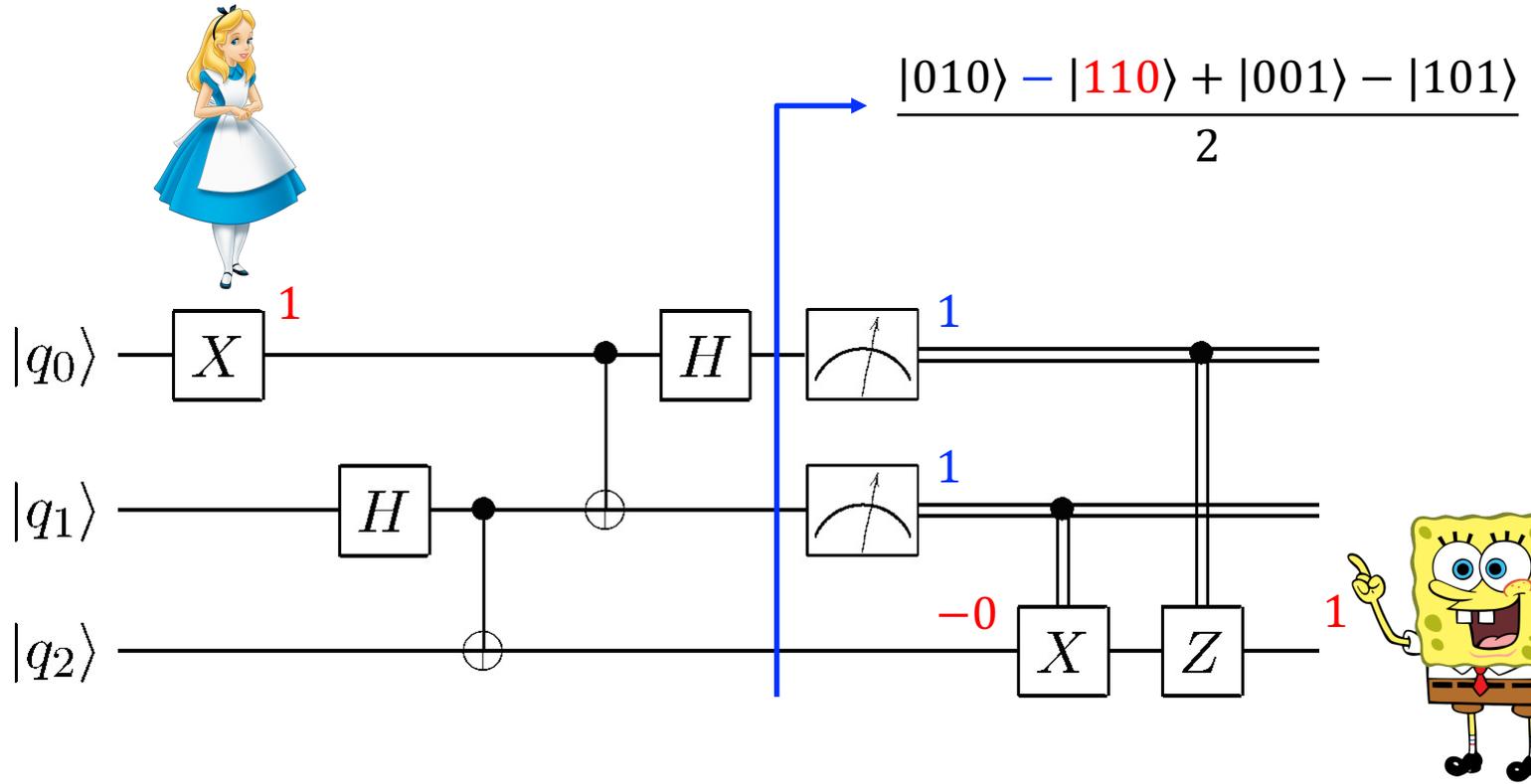
# quantum teleportation



# quantum teleportation



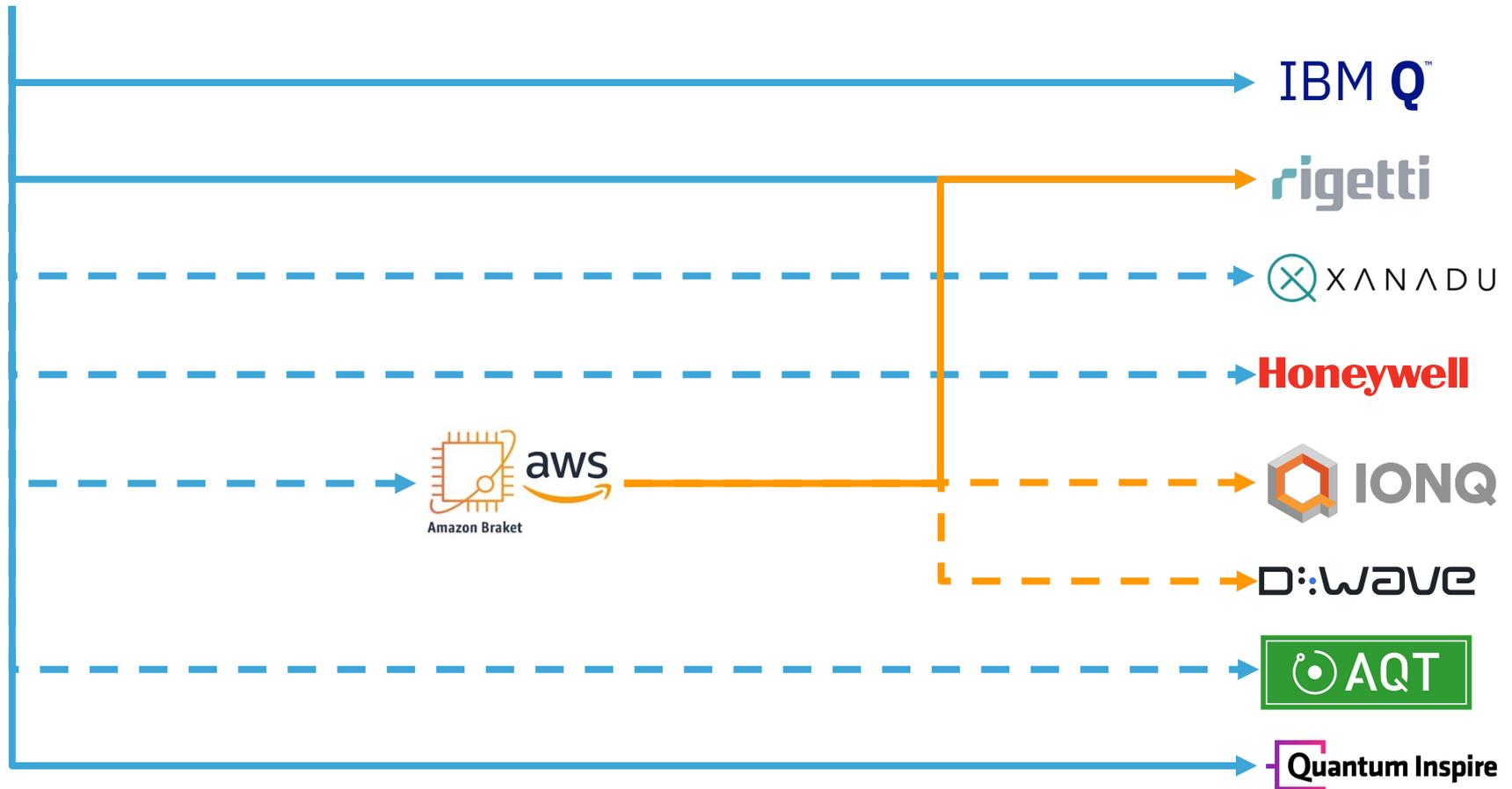
# quantum teleportation



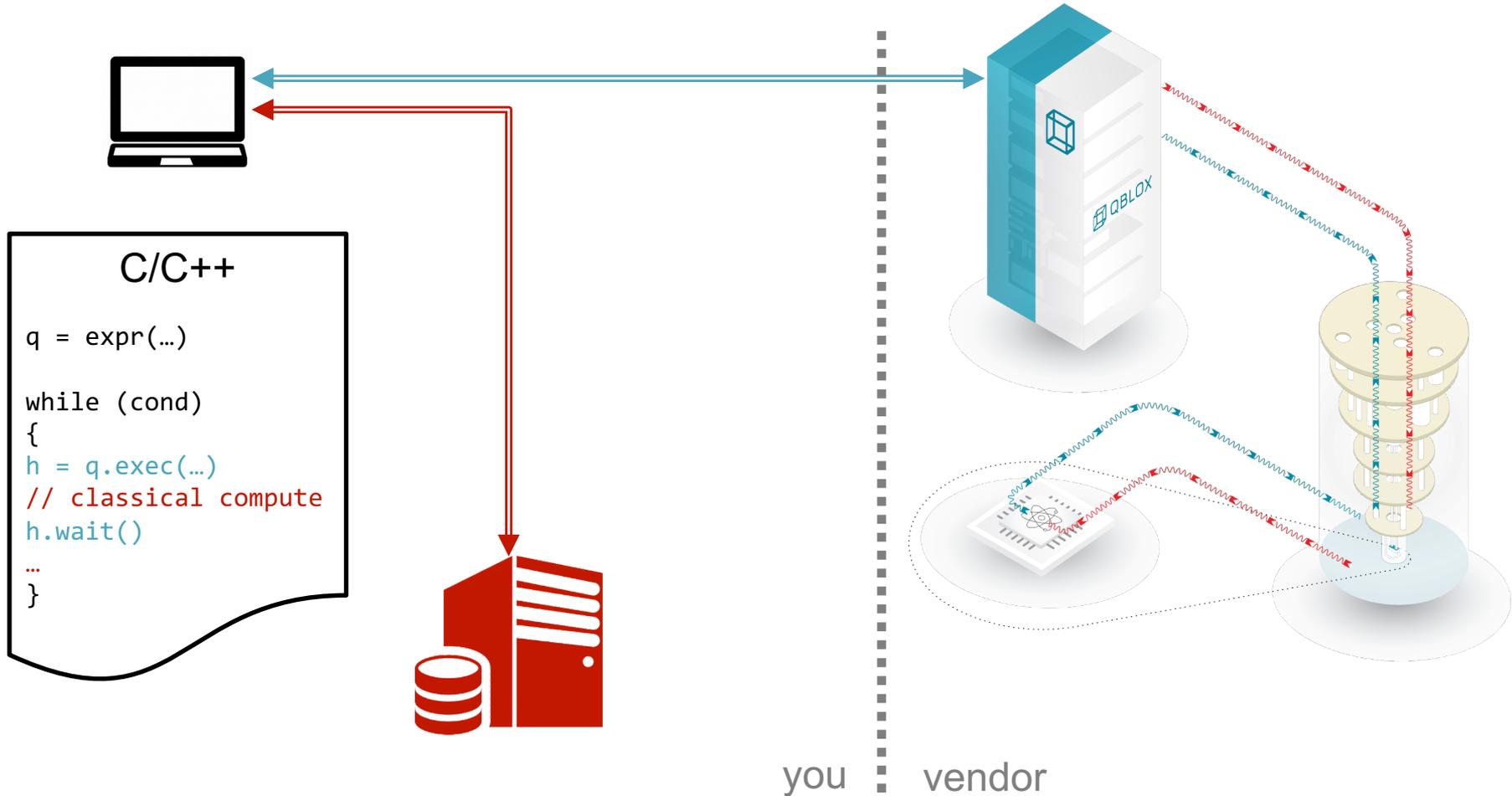
# programming tools

*LibKet and generation of resource-optimal quantum circuits*

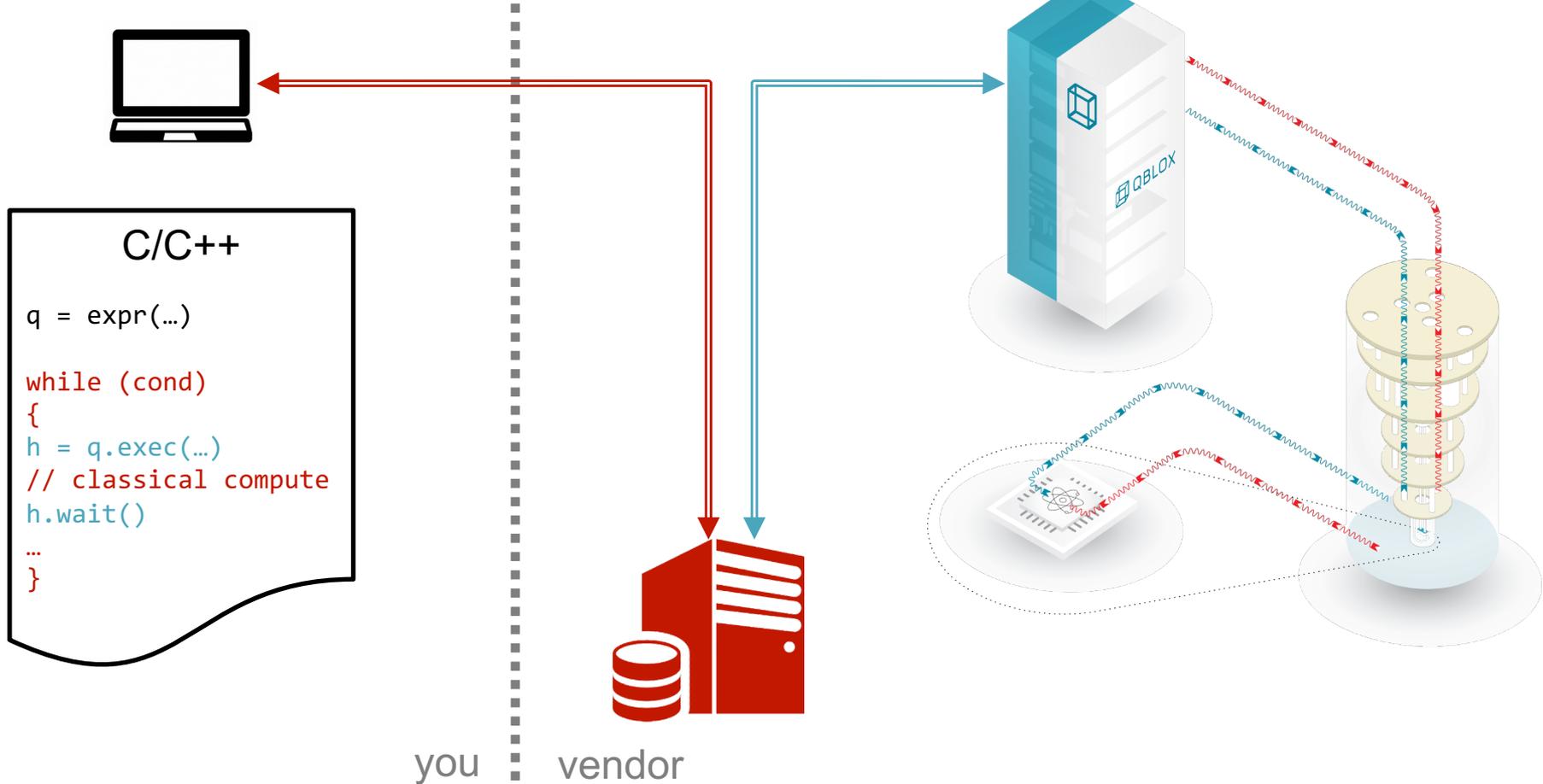
# **Lib** – The **k**wantum **e**xpression **t**emplate **L**ibrary



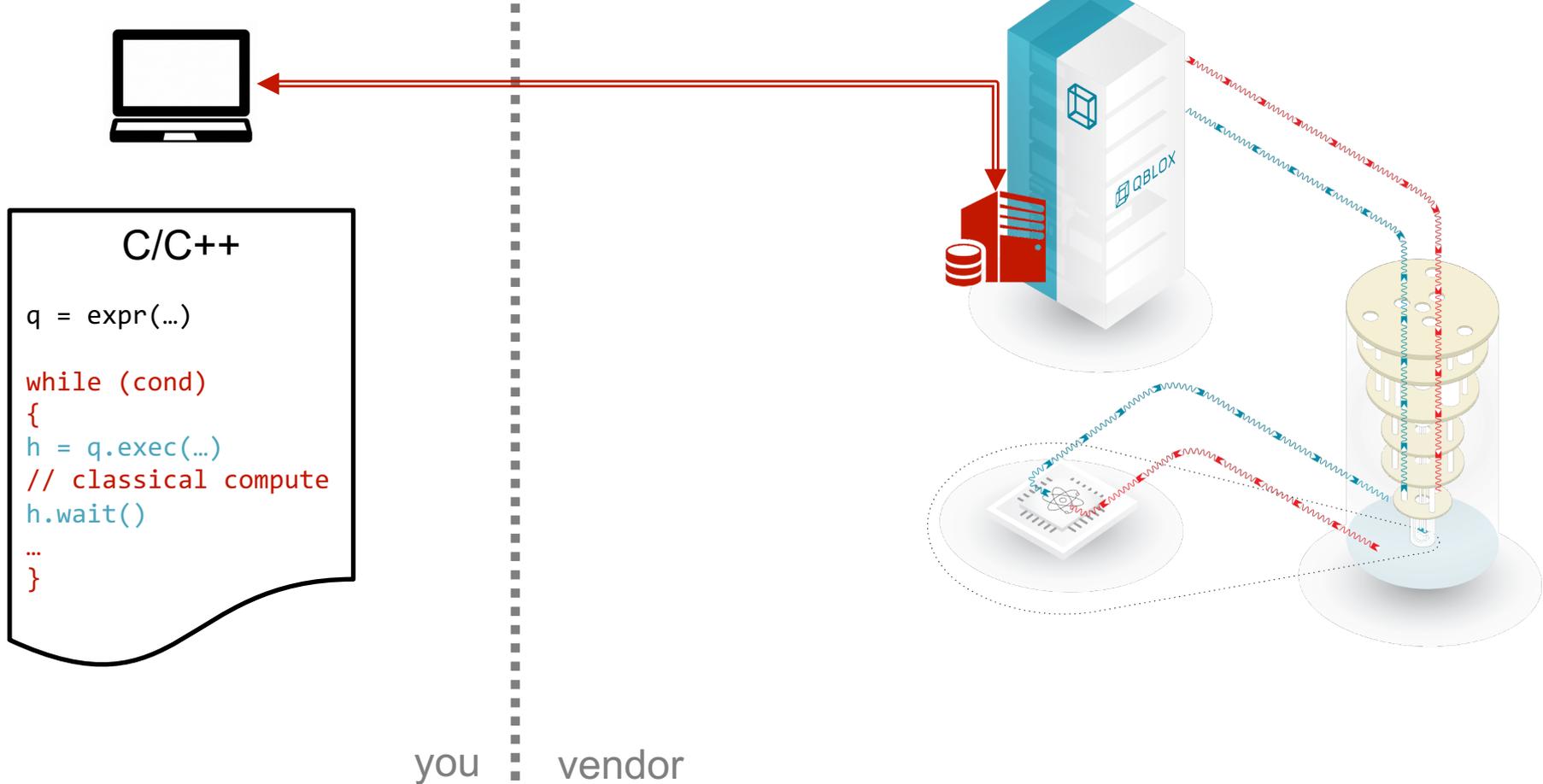
# quantum acceleration workflow



# quantum acceleration workflow



# quantum acceleration workflow



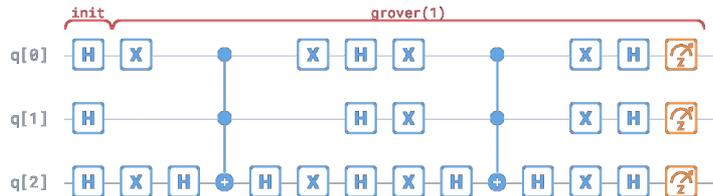
# different programming philosophies

## standard quantum SDKs

- apply gates to individual qubits

```
H    q[0:2]
X    q[0,2]
H    q[2]
CCX  q[0], q[1], q[2]
```

...



## LibKet

- 'stream' qubits through gates

```
...CCX(q[0],
        q[1],
        q[2](
            H(q[2](
                X(q[0,2](
                    H(q[0:2]())
                ))
            ))
        ))
```

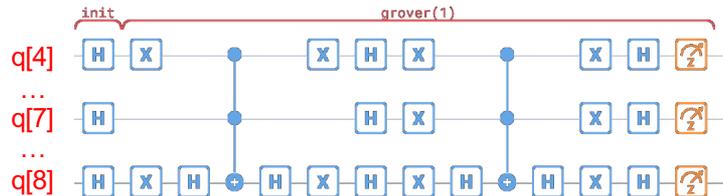
# different programming philosophies

## standard quantum SDKs

- apply gates to individual qubits

```
H    q[4,7,8]
X    q[4,8]
H    q[8]
CCX  q[4], q[7], q[8]
```

...



## LibKet

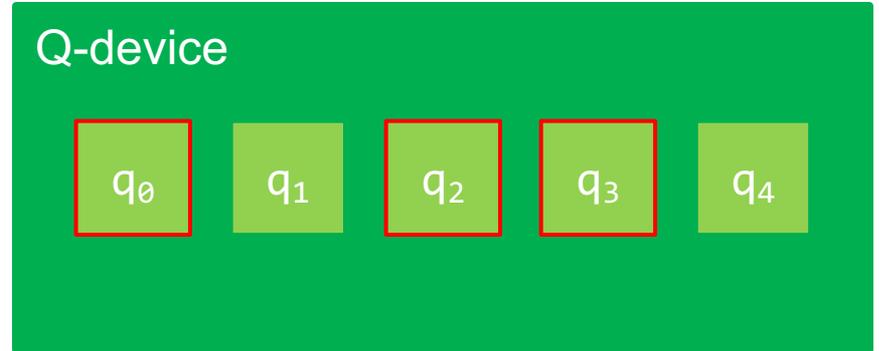
- 'stream' qubits through gates

```
...CCX(q[0],
        q[1],
        q[2](
            H(q[2](
                X(q[0,2](
                    H(q[0:2](q[4,7,8]))
                ))
            ))
        ))
```

# filters

- selective 'views' on the qubits

```
auto f0 = select<0,2,3>();
```



# filters

- selective 'views' on the qubits

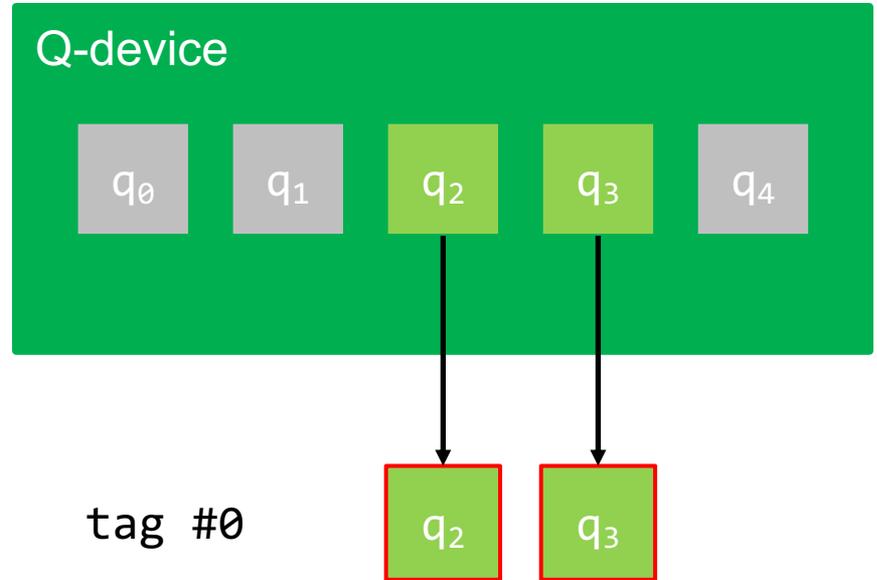
```
auto f0 = select<0,2,3>();  
auto f1 = range<1,2>(f0);
```



# filters

- selective 'views' on the qubits

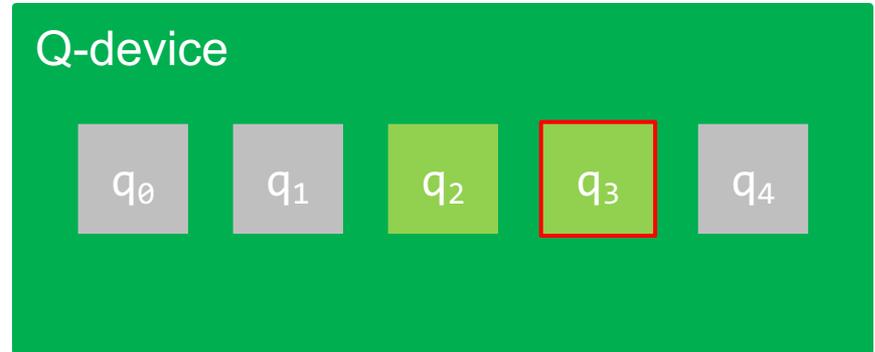
```
auto f0 = select<0,2,3>();  
auto f1 = range<1,2>(f0);  
auto f2 = tag<0>(f1);
```



# filters

- selective 'views' on the qubits

```
auto f0 = select<0,2,3>();  
auto f1 = range<1,2>(f0);  
auto f2 = tag<0>(f1);  
auto f3 = qubit<1>(f2);
```



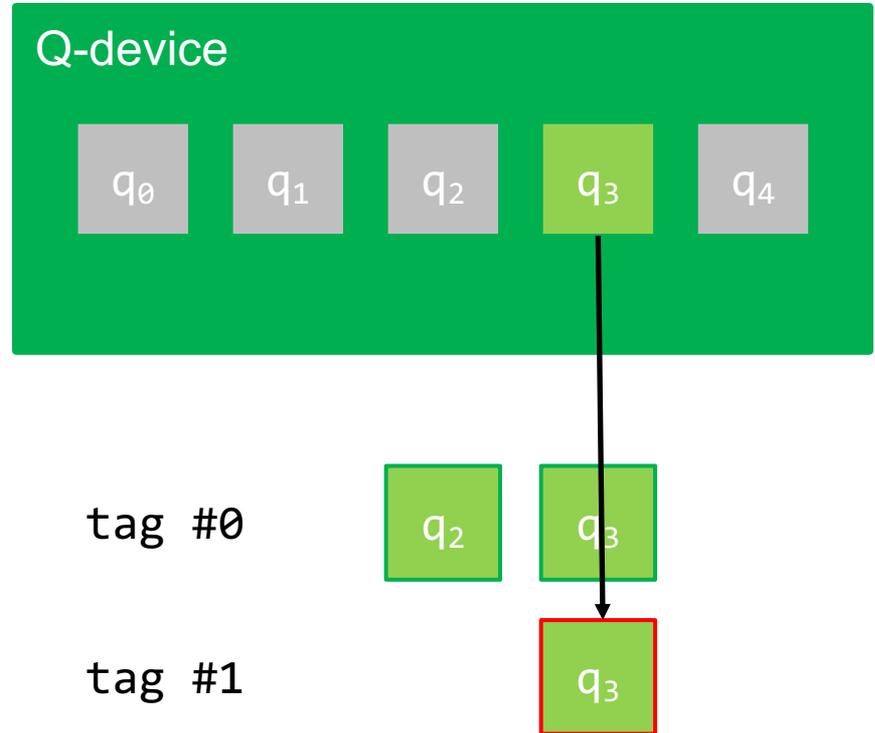
tag #0



# filters

- selective 'views' on the qubits

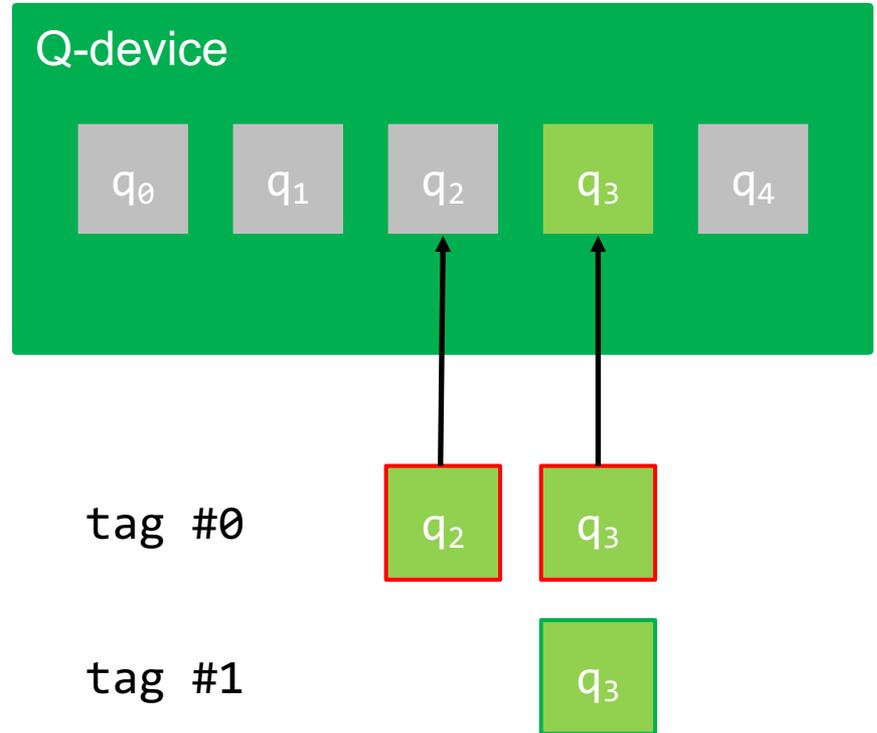
```
auto f0 = select<0,2,3>();  
auto f1 = range<1,2>(f0);  
auto f2 = tag<0>(f1);  
auto f3 = qubit<1>(f2);  
auto f4 = tag<1>(f3);
```



# filters

- selective 'views' on the qubits

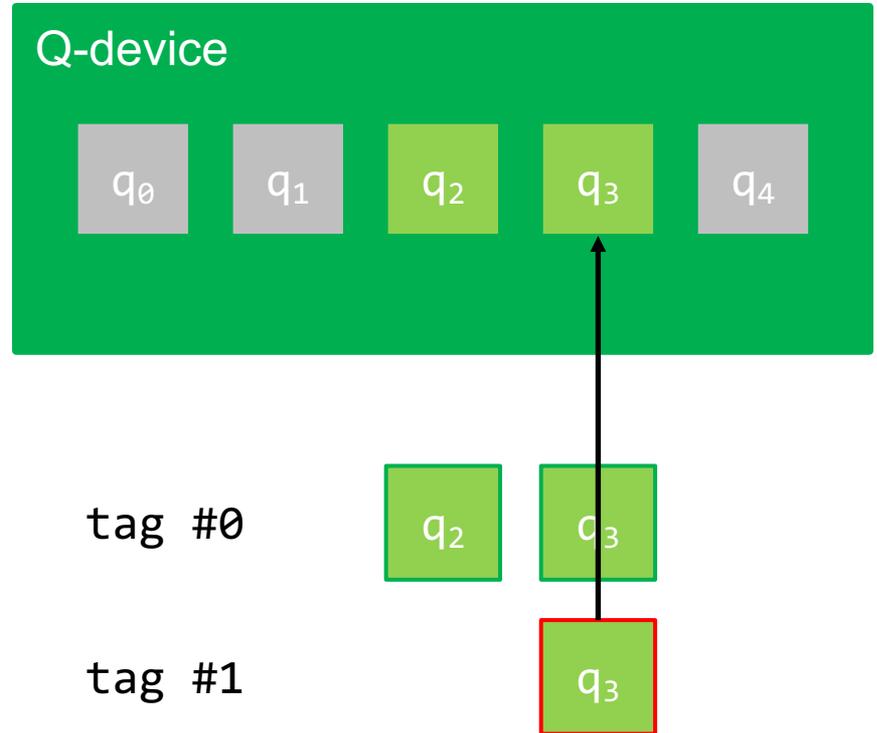
```
auto f0 = select<0,2,3>();  
auto f1 = range<1,2>(f0);  
auto f2 = tag<0>(f1);  
auto f3 = qubit<1>(f2);  
auto f4 = tag<1>(f3);  
auto f5 = gototag<0>(f4);
```



# filters

- selective 'views' on the qubits

```
auto f0 = select<0,2,3>();  
auto f1 = range<1,2>(f0);  
auto f2 = tag<0>(f1);  
auto f3 = qubit<1>(f2);  
auto f4 = tag<1>(f3);  
auto f5 = gototag<0>(f4);  
auto f6 = gototag<1>(f5);
```



# gates

- SIMD-like quantum operation on all qubits of the current filter chain

```
auto e0 = init();
```

q<sub>0</sub>

q<sub>1</sub>

q<sub>2</sub>

q<sub>3</sub>

q<sub>4</sub>

# gates

- SIMD-like quantum operation on all qubits of the current filter chain

```
auto e0 = init();  
auto e1 = sel<0,2>(e0);
```

q<sub>0</sub>

q<sub>1</sub>

q<sub>2</sub>

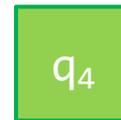
q<sub>3</sub>

q<sub>4</sub>

# gates

- SIMD-like quantum operation on all qubits of the current filter chain

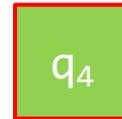
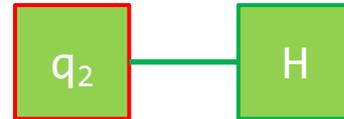
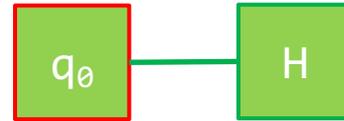
```
auto e0 = init();  
auto e1 = sel<0,2>(e0);  
auto e2 = h(e1);
```



# gates

- SIMD-like quantum operation on all qubits of the current filter chain

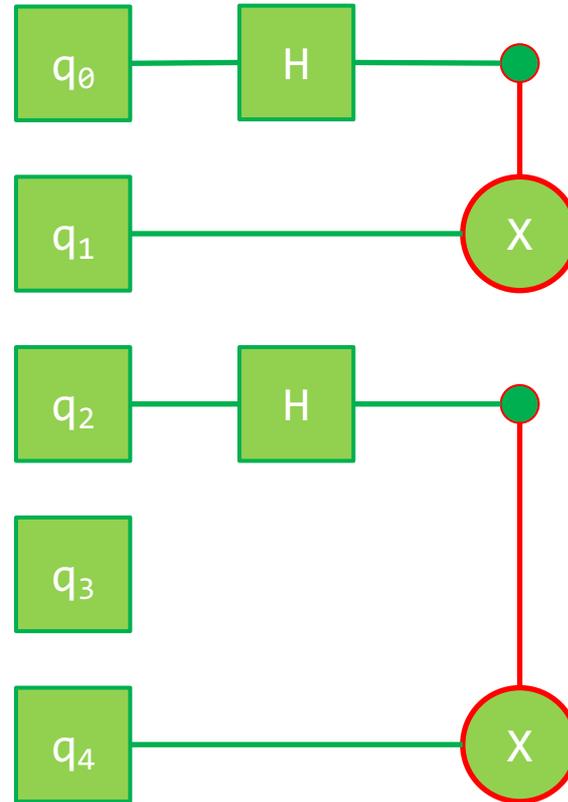
```
auto e0 = init();  
auto e1 = sel<0,2>(e0);  
auto e2 = h(e1);  
auto e3 = all(e2);
```



# gates

- SIMD-like quantum operation on all qubits of the current filter chain

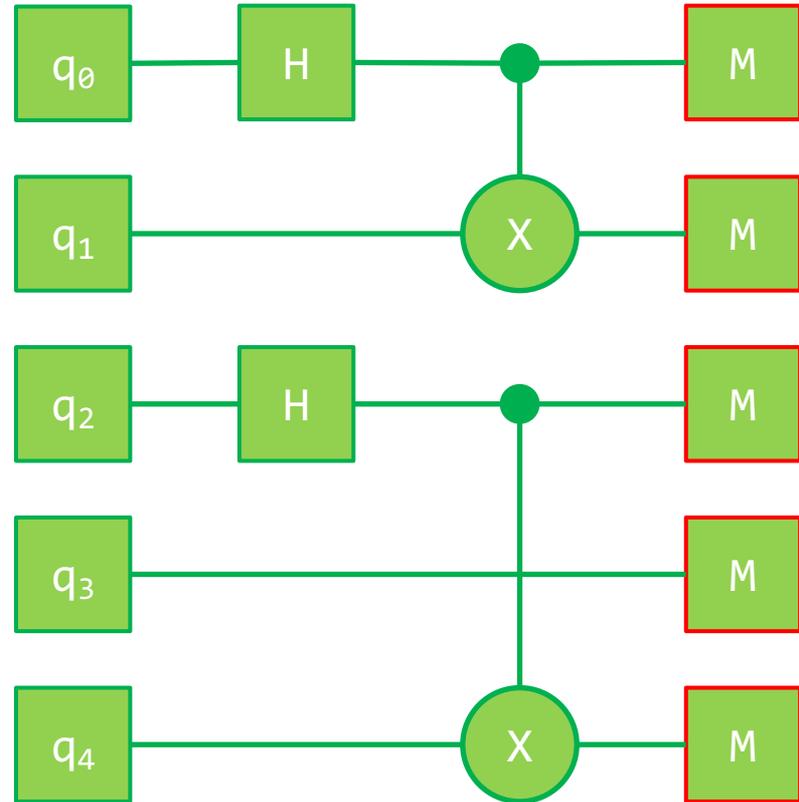
```
auto e0 = init();
auto e1 = sel<0,2>(e0);
auto e2 = h(e1);
auto e3 = all(e2);
auto e4 = cnot(
    sel<0,2>(),
    sel<1,4>(e3)
);
```



# gates

- SIMD-like quantum operation on all qubits of the current filter chain

```
auto e0 = init();
auto e1 = sel<0,2>(e0);
auto e2 = h(e1);
auto e3 = all(e2);
auto e4 = cnot(
    sel<0,2>(),
    sel<1,4>(e3)
);
auto e5 = measure(all(e4));
```

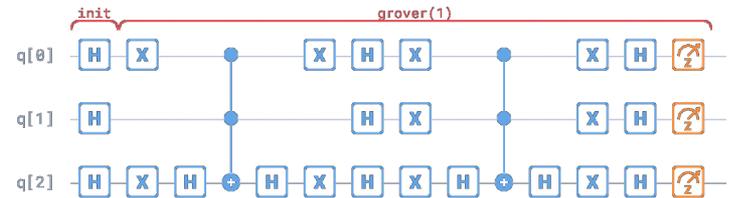


# 3-qubit Grover's algorithm

```
auto oracle = [](auto expr) {  
    return x(sel_<0>(x(h(sel_<2>(ccnot(sel_<0>(),  
                                     sel_<1>(),  
                                     sel_<2>(h(x(sel_<2>(x(sel_<0>(expr)))))))))))); };
```

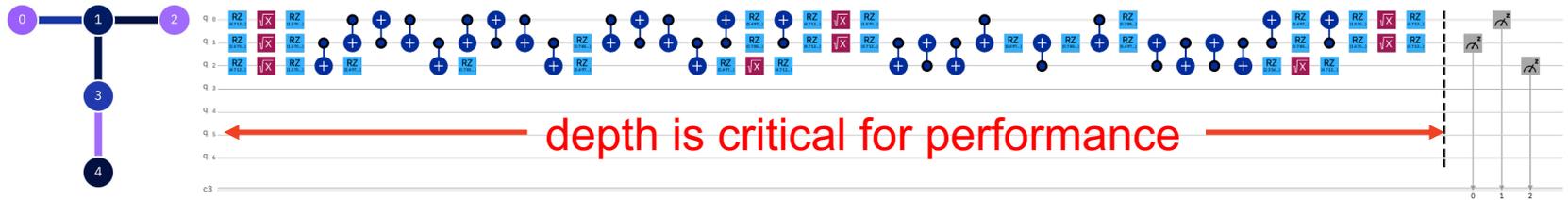
```
auto diffusion = [](auto expr) {  
    return h(x(all(h(sel_<2>(ccnot(sel_<0>(),  
                                   sel_<1>(),  
                                   sel_<2>(h(sel_<2>(x(h(all(expr)))))))))))); };
```

```
auto expr = measure(diffusion(oracle(h(init()))));  
QDevice<backend, 3> device;  
utils::json res = device(expr).eval(shots);  
cout << device.get<QResultType::best>(res) << endl;
```



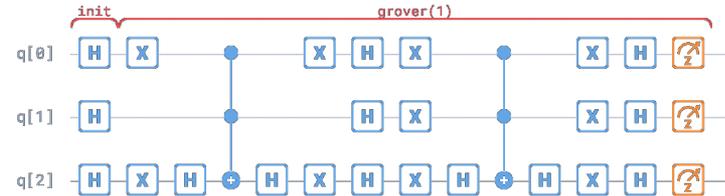
# 3-qubit Grover's algorithm

- IBM's basis gates: CX, ID, RZ, SX, X
- executable* quantum circuit generated by IBM's quantum compiler



```

auto expr = measure(diffusion(oracle(h(init()))));
QDevice<QDeviceType::ibmq_quito, 3> device;
utils::json res = device(expr).eval(shots);
cout << device.get<QResultType::best>(res) << endl;
    
```



# traditional quantum circuit compilation

- gate substitution rules

$$H \rightarrow R_x(\pi)R_y(\pi/2), \quad H \rightarrow R_y(-\pi/2)R_x(\pi), \quad \dots$$

- cancelling of inverse gates

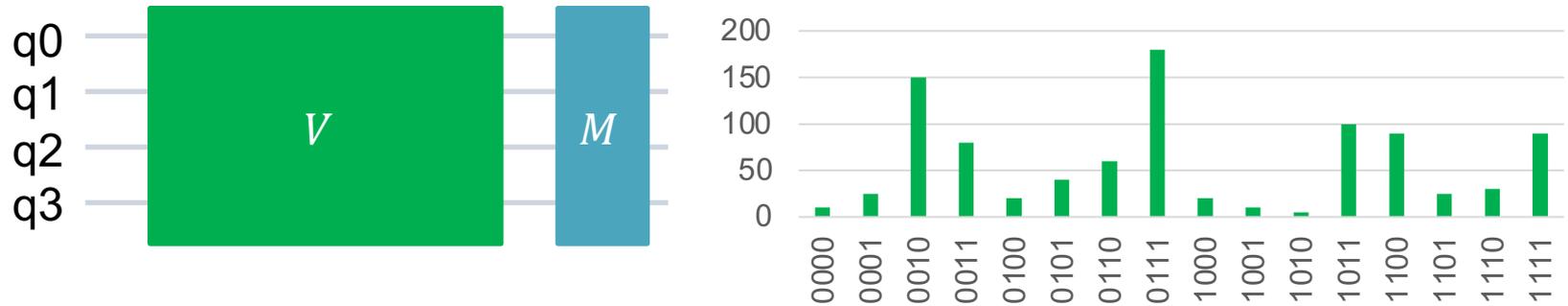
$$CZ CZ^\dagger = I, \quad R_x(\theta)R_x(-\theta) = I, \quad \dots$$

- aggregation using commutativity or fusion rules

$$HR_z(\theta)H = R_x(\theta), \theta \in \{\pi, \pm \pi/2\}, \quad R_z(\theta_1)R_z(\theta_2) = R_z(\theta_1 + \theta_2), \quad \dots$$

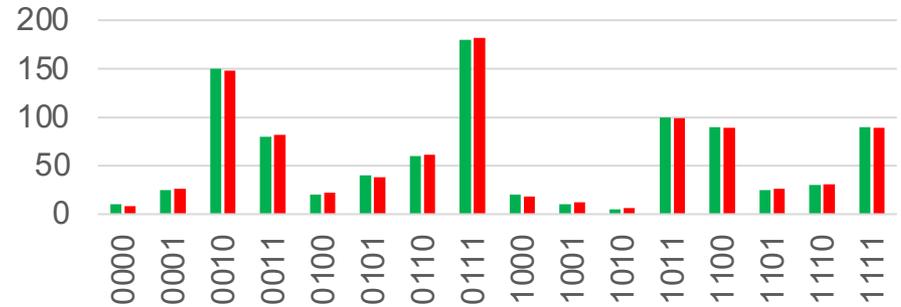
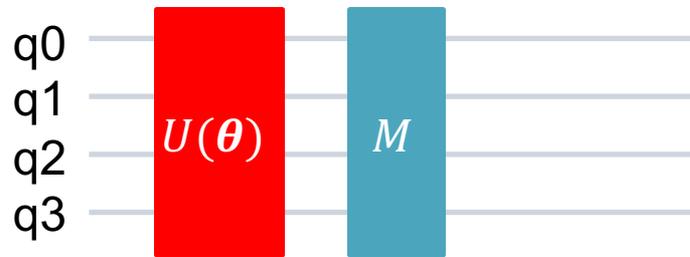
# approximate computing

- our aim is to generate a resource-efficient directly executable circuit  $U(\theta)$  that mimics the expectation-value behavior of the textbook circuit  $V$



# approximate computing

- our aim is to generate a resource-efficient directly executable circuit  $U(\theta)$  that mimics the expectation-value behavior of the textbook circuit  $V$



# approximate computing

$$U_{\text{opt}} = \underset{U \in \mathcal{U}_s}{\text{argmin}} \min_{\boldsymbol{\theta}_U} \max_{|\psi\rangle \in \Psi} F(|\psi\rangle; V, U(\boldsymbol{\theta}_U)), \quad s \rightarrow \min$$

- cost function

$$F(|\psi\rangle; V, U(\boldsymbol{\theta}_U)) = \sum_k \left( \langle A^k \rangle_{V|\psi\rangle} - \langle A^k \rangle_{U(\boldsymbol{\theta}_U)|\psi\rangle} \right)$$

- $A^k$  is an observable, e.g., Pauli-X, Y, Z gate
- expectation value of state  $|\psi\rangle$  upon application of operator  $P$

$$\langle A^k \rangle_{P|\psi\rangle} = \langle (P\psi)^\dagger | A^k | P\psi \rangle$$

- $\mathcal{U}_s$  is the set of all admissible quantum circuits of size  $s$
- $U(\boldsymbol{\theta}_U)$  is one parametrized quantum circuit with  $\boldsymbol{\theta}_U = (\theta_1, \dots, \theta_N)^\top$

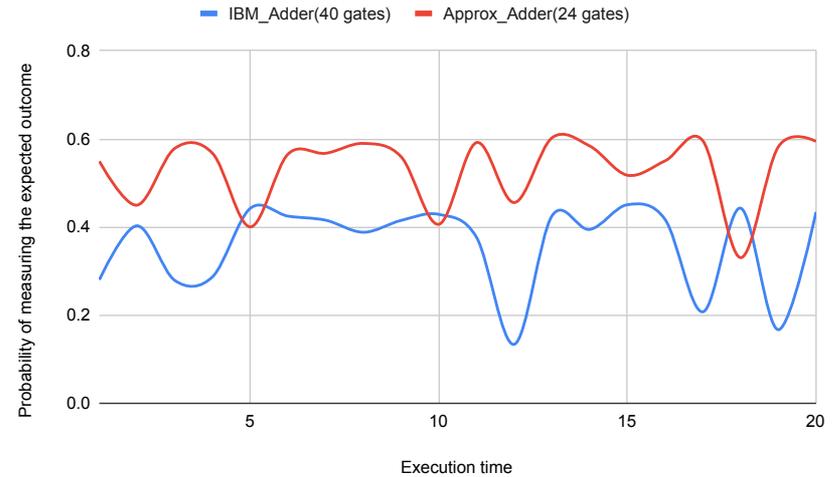
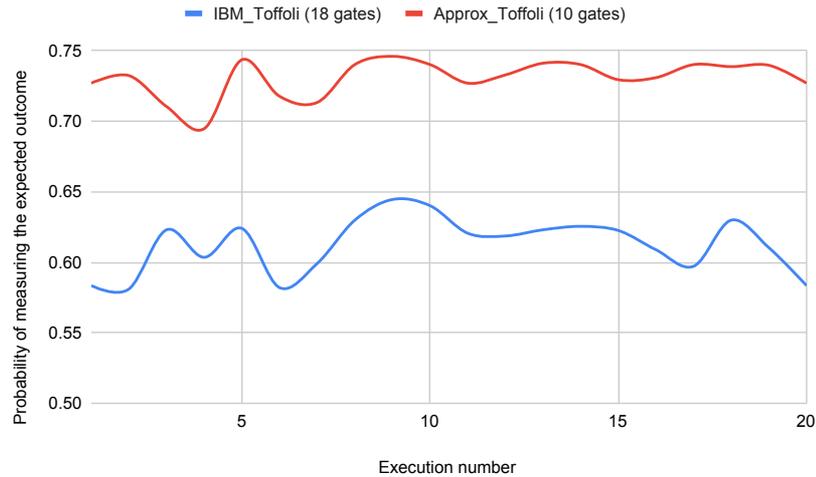
# selected results



algo	#qubits	rigetti	ours		ibm	ours	
2q qft	2	18	16	11%	12-42	11	8-73%
3q qft	5	118	92	22%	57-73	56	1-23%
ccnot	3	33	15	54%	18	10	44%
4q add	8	197	132	33%	116-131	74	36-43%
8q add	16	474	312	34%	272-299	174	36-42%
mc3x	4	90	76	15%	46-102	36	21-64%
mc4x	5	195	164	16%	94-150	76	23-49%
2q grover	2	15	8	46%	16-27	8	50-71%
bv	4	21	11	47%	22	11	50%

S. Adarsh, M. Möller: Resource Optimal Executable Quantum Circuit Generation Using Approximate Computing. To appear the Proceedings of IEEE International Conference on Quantum Computing and Engineering (QCE21), 2021.

# selected results



S. Adarsh, M. Möller: Resource Optimal Executable Quantum Circuit Generation Using Approximate Computing. To appear the Proceedings of IEEE International Conference on Quantum Computing and Engineering (QCE21), 2021.

# applications

*quantum linear solvers and optimization algorithms*

# potential quantum applications

$$\begin{aligned} \min_{\theta} & x_{\theta}^{\dagger} M_{\theta} x_{\theta} \\ \text{s. t.} & A_{\theta} x_{\theta} = b_{\theta} \end{aligned}$$

## ▪ HHL-type quantum linear solver

$$\text{Find } x^{\dagger} M x \quad \text{s. t. } Ax = b$$

- sparse matrices  $O(\log(N)\kappa^2/\epsilon)$  [Harrow, Hassidim, Lloyd 2009]  
 $\text{polylog}(1/\epsilon)$  [Childs, Kothari, Somma 2017]
- dense matrices  $O(\sqrt{N} \log(N)\kappa^2/\epsilon)$  [Wossnig et al. 2018]

## ▪ applications

- linear differential equations [Berry 2010, Xin et al. 2018]
- nonlinear differential equations [Leyton, Osborne 2008, Liu et al. 2021]
- Poisson equation [Cao et al. 2013, Montanaro 2015]
- principal component analysis [Lloyd et al. 2014]
- data fitting [Wiebe et al. 2012]
- machine learning [Lloyd et al. 2013, Adcock et al. 2015, Biamonte et al. 2017, Schuld et al. 2018, Perdomo-Ortiz et al. 2018, ...]

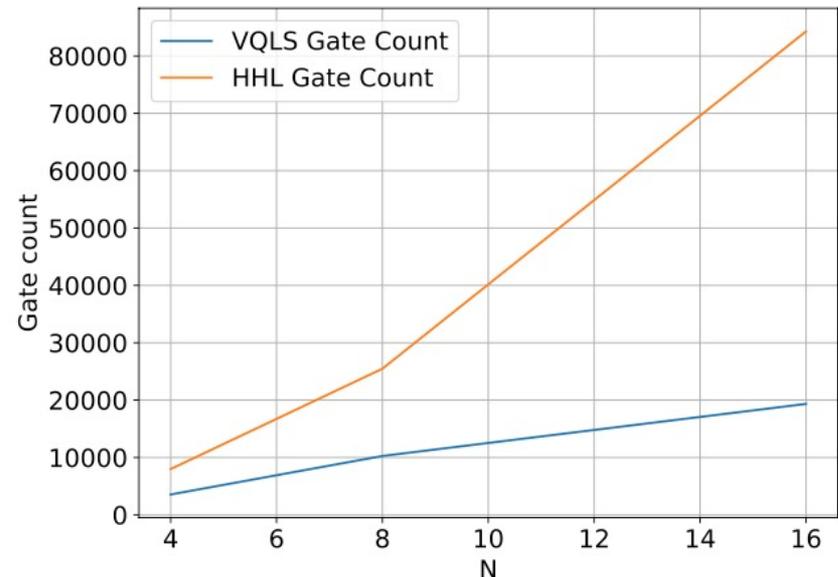


## caveats

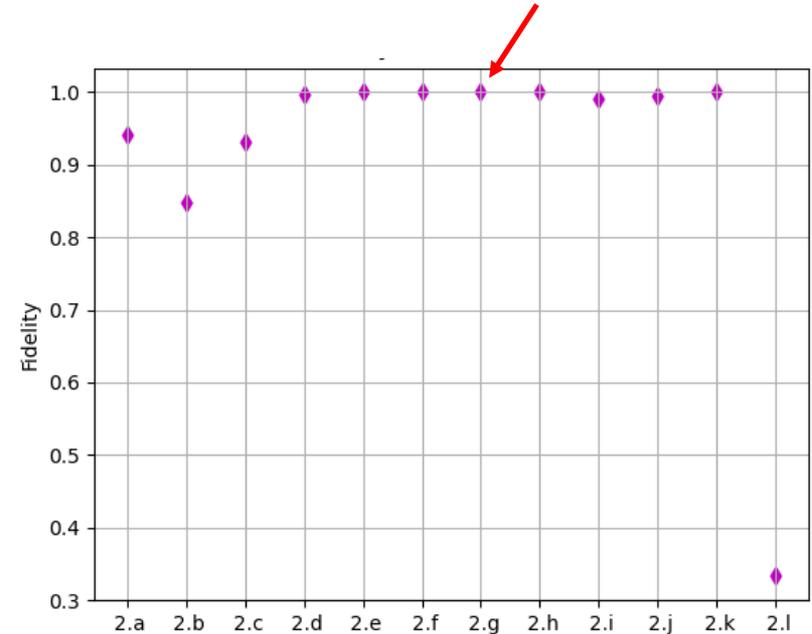
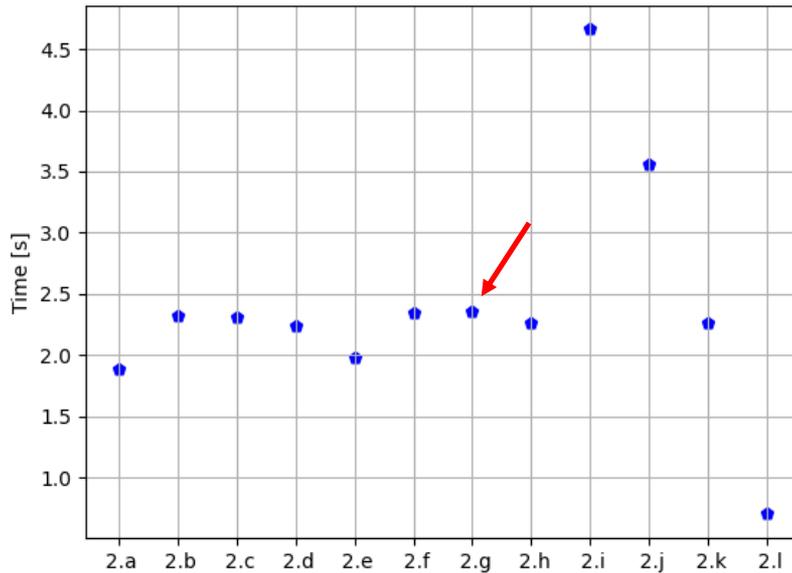
- you don't get the solution vector  $x$  but a scalar value  $x^\dagger M x$
- circuits are impractical for near-future quantum computers
- Recent step-by-step HHL algorithm walk-through by Morrell and Wong (08/2021):

*"[...] due to the imperfection and noise in a real quantum computer (ibmq\_santiago), the hardware execution of the same circuit (for a 2x2 matrix) does not give satisfactory result"*

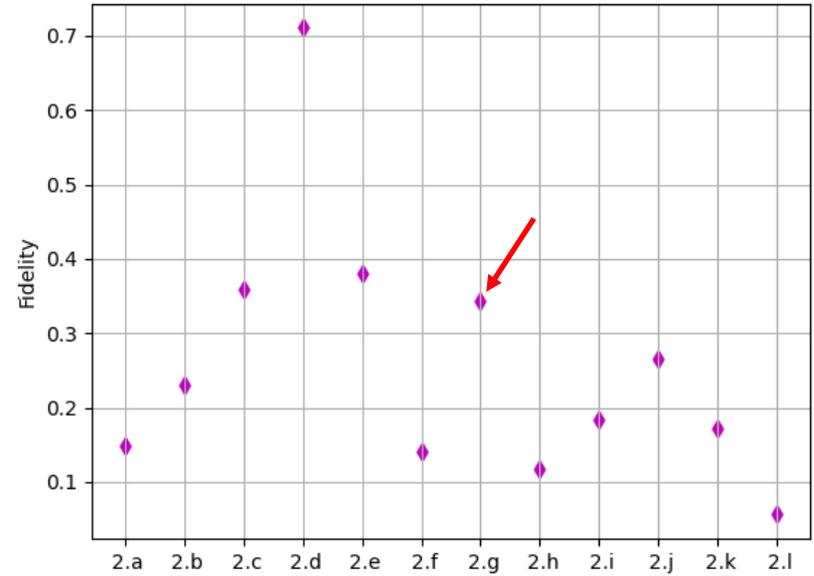
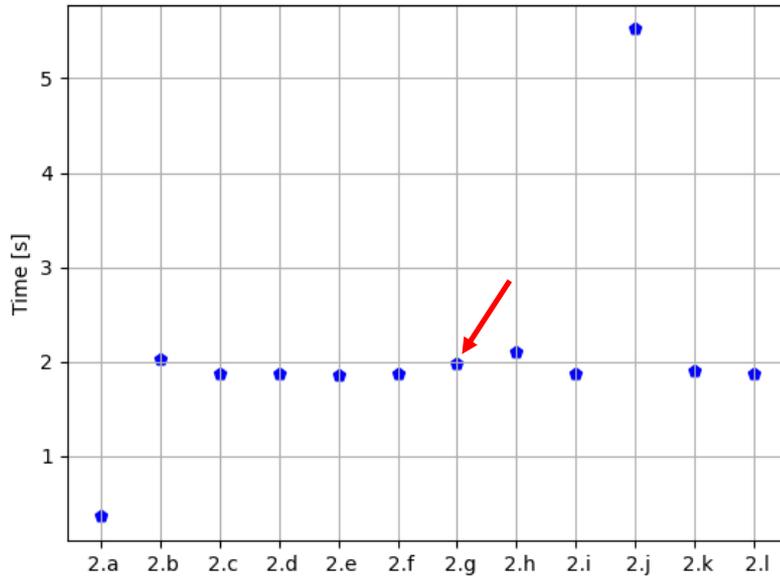
arXiv:2108.09004



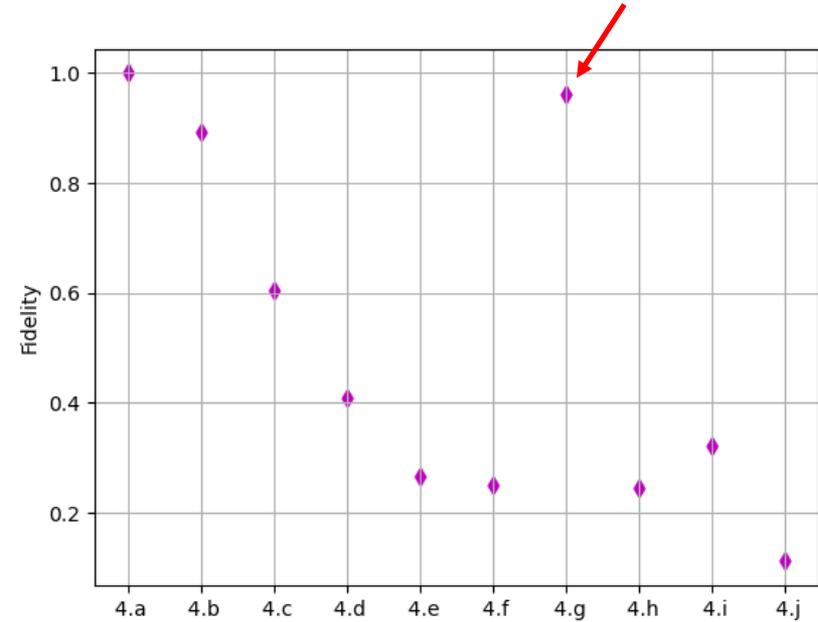
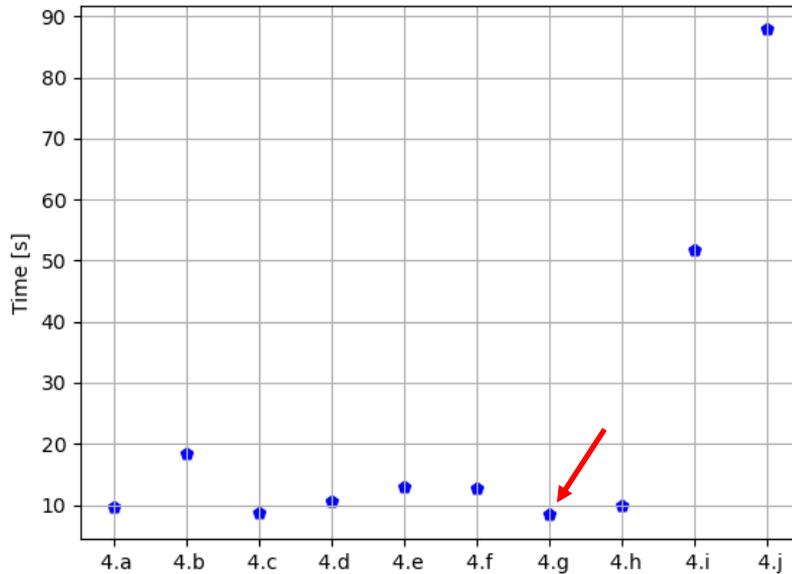
# HHL simulation with Qiskit: 2x2 matrices, w/o noise



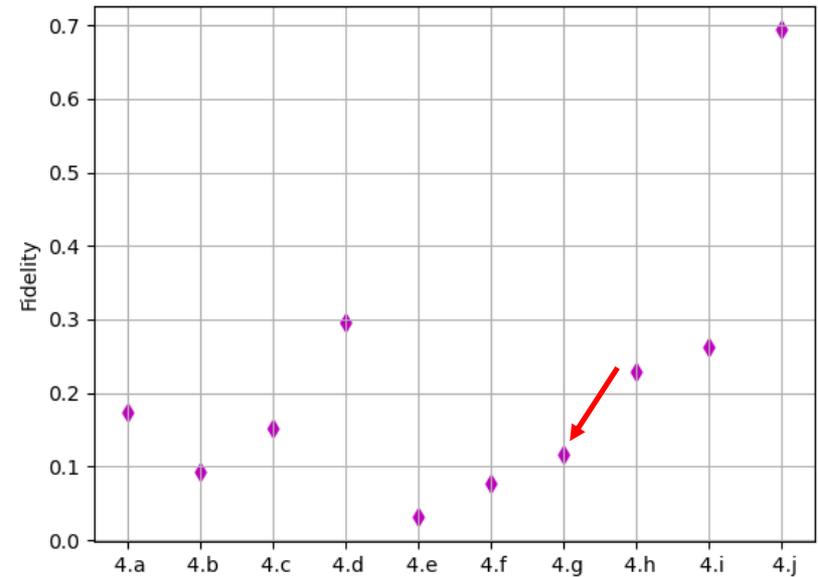
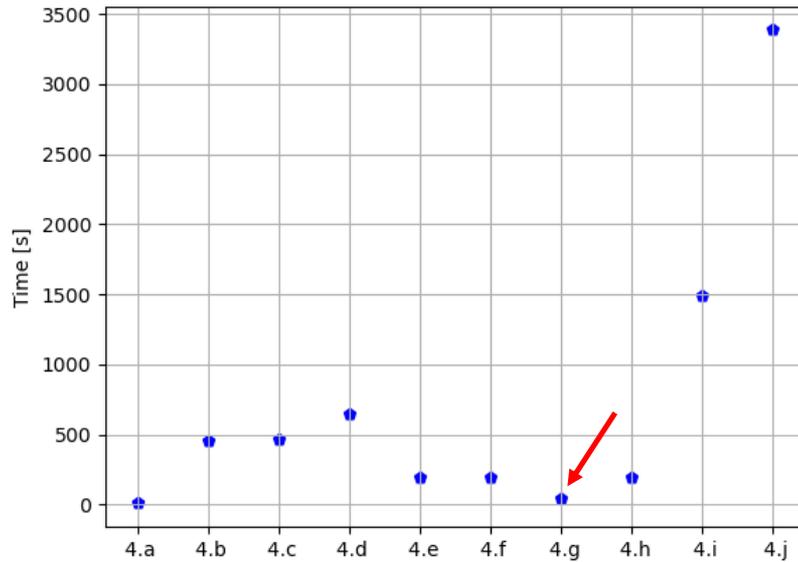
# HHL simulation with Qiskit: 2x2 matrices, with noise



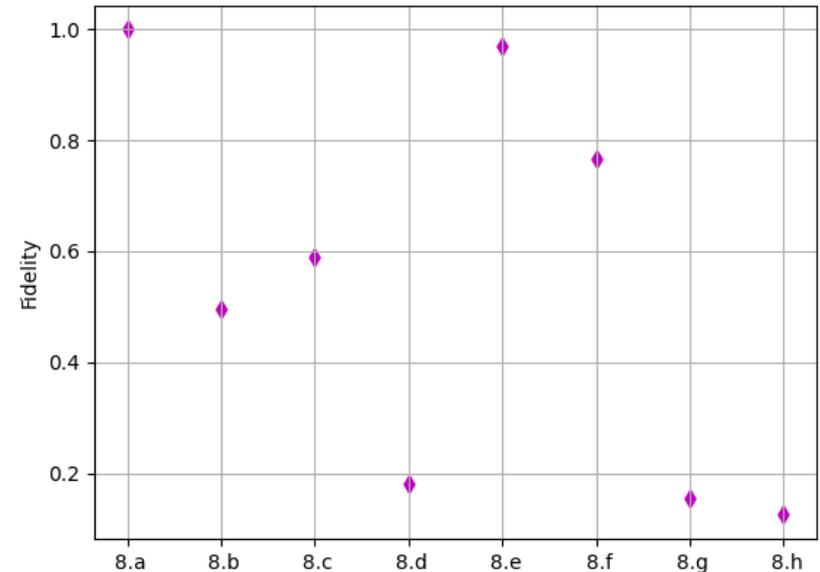
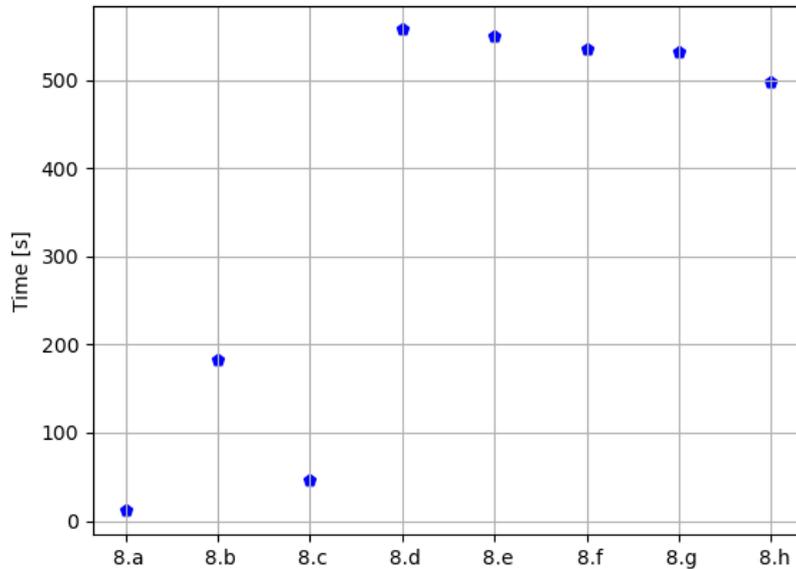
# HHL simulation with Qiskit: 4x4 matrices, w/o noise



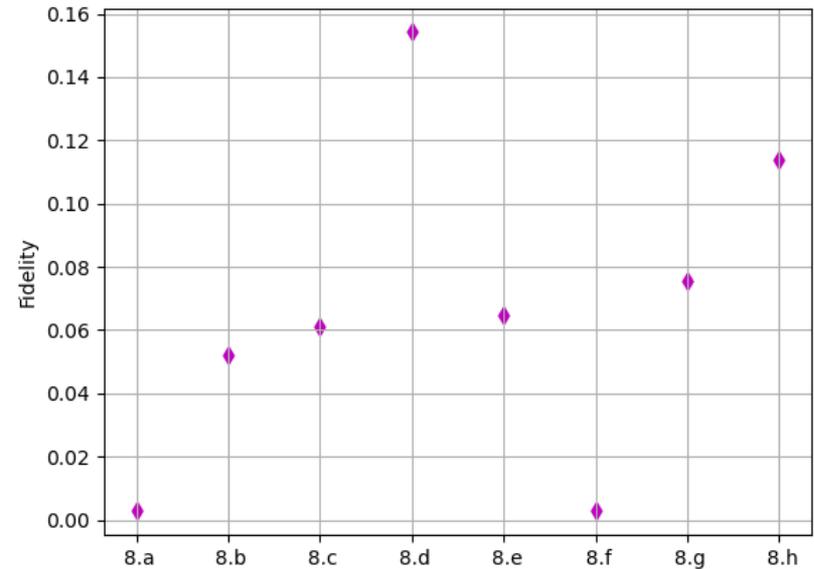
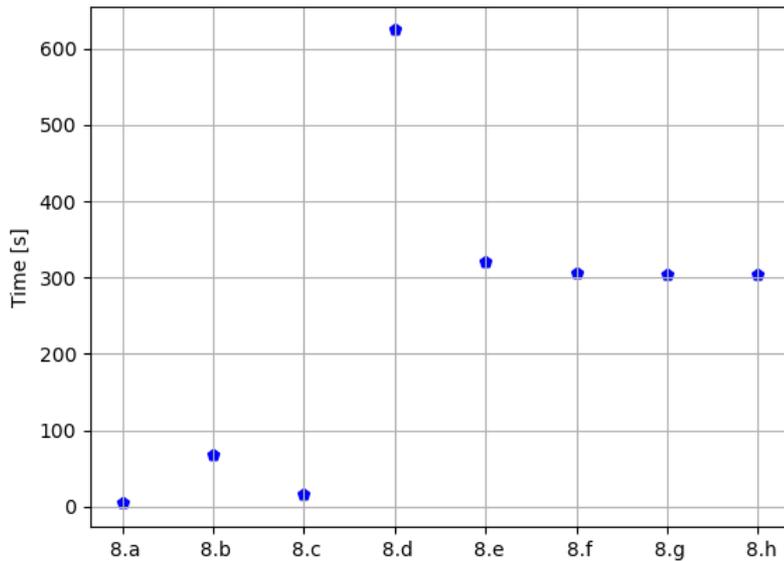
# HHL simulation with Qiskit: 4x4 matrices, with noise



# HHL simulation with Qiskit: 8x8 matrices, w/o noise



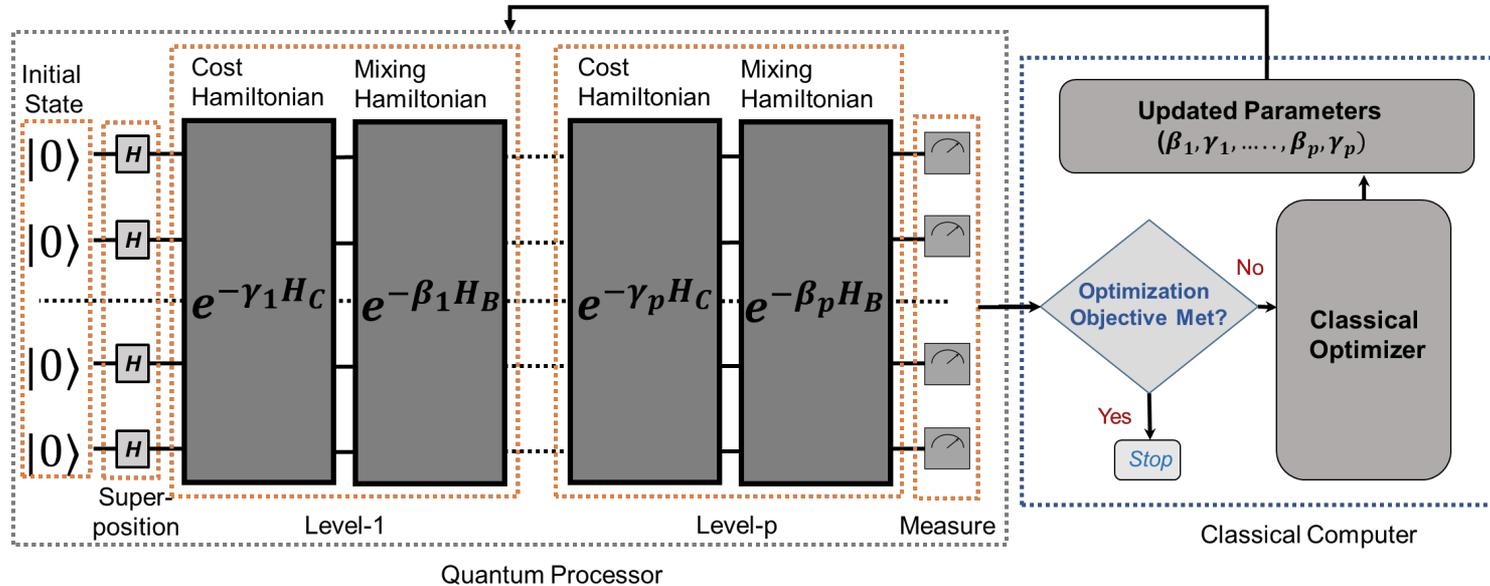
# HHL simulation with Qiskit: 8x8 matrices, with noise



## potential *near-future* quantum applications in SciComp

- **hybrid quantum-classical algorithms**
  - quantum approximate optimization algorithm (QAOA) [Farhi et al. 2014]
  - quantum alternating operator ansatz (QAOA) [Hadfield et al. 2017]
  - variational quantum eigensolver (VQE) [Peruzzo et al. 2014]
  - variational quantum linear solver (VQLS) for sparse matrices [Bravo-Prieto et al. 2019 & Xu et al. 2019]

# QAOA workflow



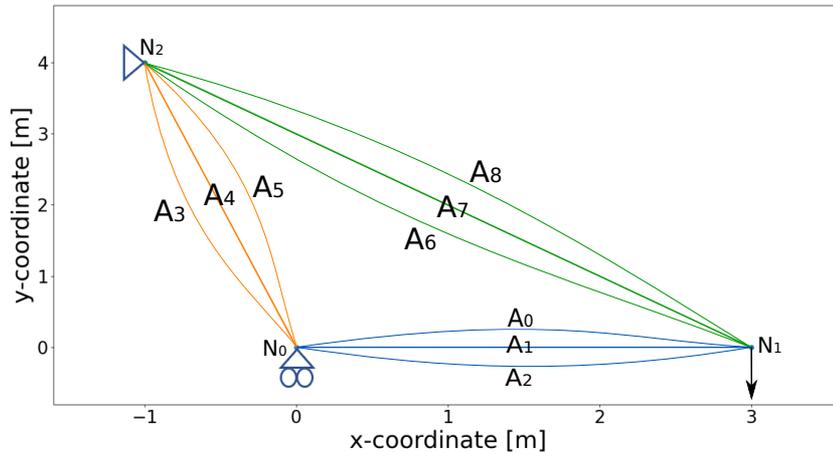
M. Alam, A. Ash-Saki, S. Ghosh: Analysis of quantum approximate optimization algorithm under realistic noise in superconducting qubits. arXiv: 1907.09631 (2019)

# truss structure optimization



# 3-truss structure

options:  $2^{\text{\#trusses} \times \text{\#areas}} = 512$



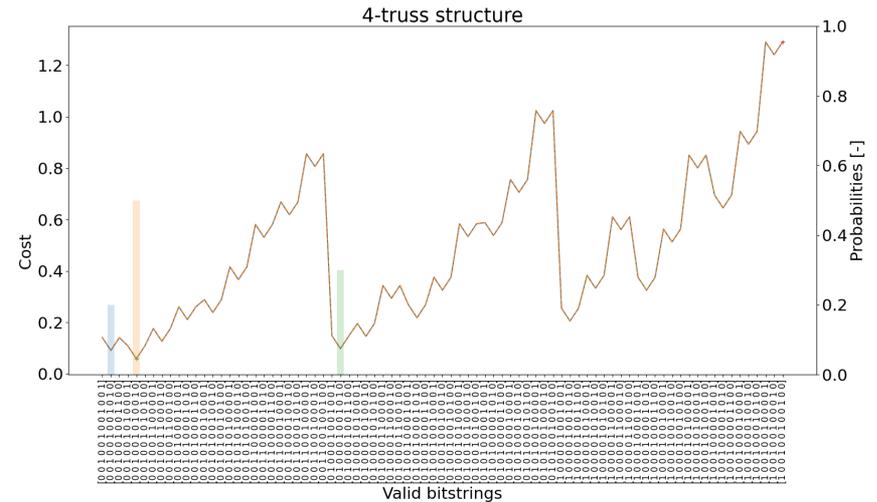
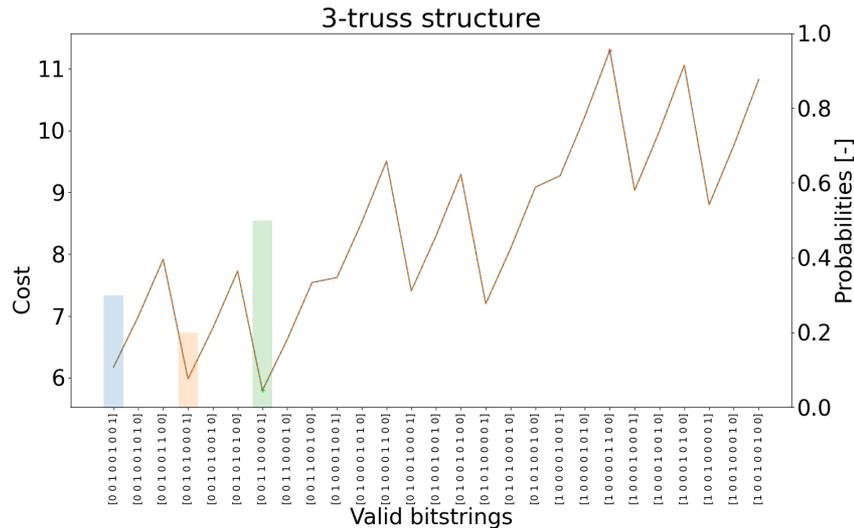
valid options

invalid options

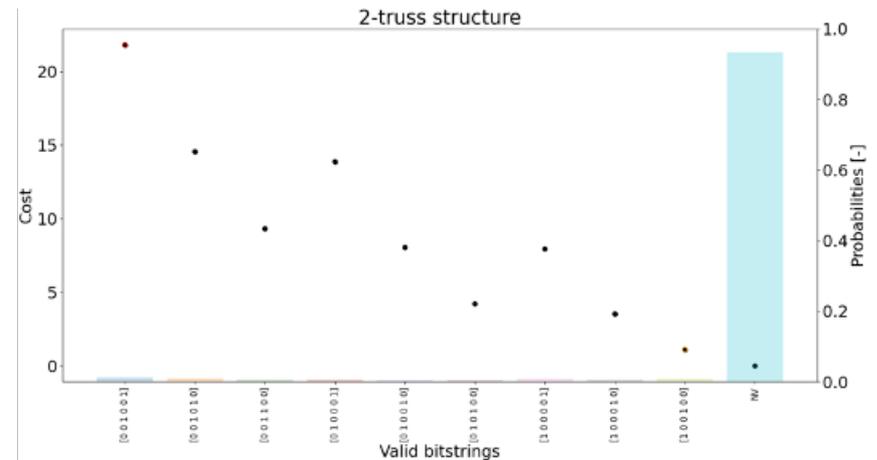
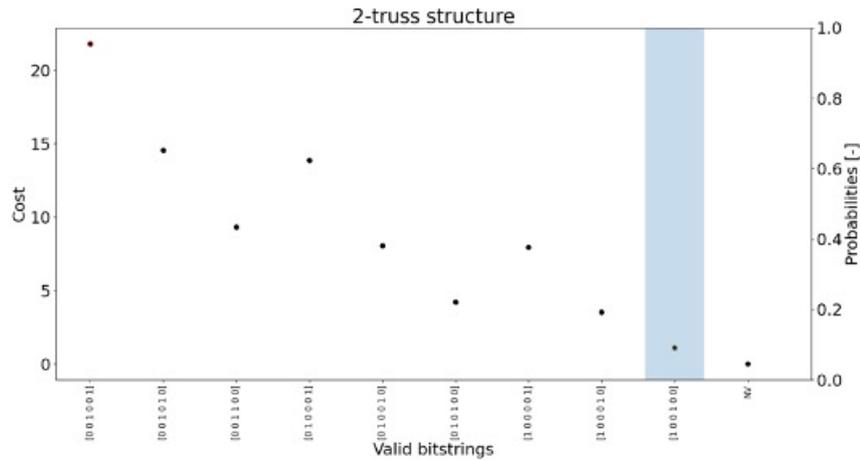
Option	$q_0$	$q_1$	$q_2$	$q_3$	$q_4$	$q_5$	$q_6$	$q_7$	$q_8$
1	0	0	1	0	0	1	0	0	1
2	0	0	1	0	0	1	0	1	0
3	0	0	1	0	0	1	1	0	0
4	0	0	1	0	1	0	0	0	1
5	0	0	1	0	1	0	0	1	0
6	0	0	1	0	1	0	1	0	0
7	0	0	1	1	0	0	0	0	1
8	0	0	1	1	0	0	0	1	0
9	0	0	1	1	0	0	1	0	0
10	0	1	0	0	0	1	0	0	1
11	0	1	0	0	0	1	0	1	0
12	0	1	0	0	0	1	1	0	0
13	0	1	0	0	1	0	0	0	1
14	0	1	0	0	1	0	0	1	0
15	0	1	0	0	1	0	1	0	0
16	0	1	0	1	0	0	0	0	1
17	0	1	0	1	0	0	0	1	0
18	0	1	0	1	0	0	1	0	0
19	1	0	0	0	0	1	0	0	1
20	1	0	0	0	0	1	0	1	0
21	1	0	0	0	0	1	1	0	0
22	1	0	0	0	1	0	0	0	1
23	1	0	0	0	1	0	0	1	0
24	1	0	0	0	1	0	1	0	0
25	1	0	0	1	0	0	0	0	1
26	1	0	0	1	0	0	0	1	0
27	1	0	0	1	0	0	1	0	0

485

# preliminary results using Rigetti's simulator



# preliminary results using Rigetti's Aspen-9 processor



only 9 out of 64 options are valid and the exclusion criterion is sensitive to noise

## summary

- applied quantum computing is fun
- full-quantum algorithms are not near-future
- hybrid quantum-classical algorithms also have their subtleties
- nonetheless, it's time for application-driven practical quantum computing

Thank you for your attention!